# EMF IN A SPINNING SPHERICAL SHELL

Link to: physicspages home page.
To leave a comment or report an error, please use the auxiliary blog.
References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.45.

Here's a simple example of calculating the emf directly from its definition as the work done on a unit charge over a path.

$$\mathscr{E}=\oint\mathbf{f}\cdot d\ell \tag{1}$$

Usually, the integral is taken around a closed loop such as an electric circuit, but we can use the same definition to calculate the emf between two points on a surface. Suppose we have a superconducting spherical shell of radius $a$ that is spinning on its axis (taken to be the $z$ axis) with angular speed $\omega$. We impose a constant, uniform magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$. If we look at an infinitesimal horizontal slice through the sphere taken at a polar angle of $\theta$, we can think of this slice as a circular loop carrying a current. If the sphere is spinning in the $+\phi$ direction (counterclockwise seen from above), then the magnetic force per unit charge is

$$\begin{aligned}
\mathbf{f} &= \mathbf{v}\times\mathbf{B} && (2)\\
&= \omega aB\sin\theta\hat{\phi}\times\hat{\mathbf{z}} && (3)
\end{aligned}$$

The vector $\hat{\phi}\times\hat{\mathbf{z}}$ points horizontally outward. If we now want to find the emf induced by the field between the north pole (at $\theta=0$) and the equator (at $\theta=\frac{\pi}{2}$), we can do the integral above over a path along a line of longitude between those two angles. Along this path

$$d\ell=a(d\theta)\hat{\theta} \tag{4}$$

The angle between $\hat{\phi}\times\hat{\mathbf{z}}$ and $\hat{\theta}$ is also $\theta$ so

$$\mathcal{E} = \oint \mathbf{f}\cdot d\ell \tag{5}$$

$$= \omega a^2 B \int_0^{\pi/2} \sin\theta \left(\hat{\phi}\times\hat{\mathbf{z}}\right)\cdot\hat{\theta} \tag{6}$$

$$= \omega a^2 B \int_0^{\pi/2} \sin\theta\cos\theta d\theta \tag{7}$$

$$= \frac{1}{2}\omega a^2 B \int_0^{\pi/2} \sin 2\theta d\theta \tag{8}$$

$$= \frac{1}{2}\omega a^2 B \tag{9}$$

Note that this is the same emf as that produced in a flat, circular disk of radius $a$ rotating at angular speed $\omega$ (Griffiths, example 7.4) so the sphere problem effectively reduces to the disk problem by projecting the sphere onto its equatorial plane.