

EMF IN A SPINNING SPHERICAL SHELL

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.45.

Here's a simple example of calculating the emf directly from its definition as the work done on a unit charge over a path.

$$\mathcal{E} = \oint \mathbf{f} \cdot d\boldsymbol{\ell} \quad (1)$$

Usually, the integral is taken around a closed loop such as an electric circuit, but we can use the same definition to calculate the emf between two points on a surface. Suppose we have a superconducting spherical shell of radius a that is spinning on its axis (taken to be the z axis) with angular speed ω . We impose a constant, uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. If we look at an infinitesimal horizontal slice through the sphere taken at a polar angle of θ , we can think of this slice as a circular loop carrying a current. If the sphere is spinning in the $+\phi$ direction (counterclockwise seen from above), then the magnetic force per unit charge is

$$\mathbf{f} = \mathbf{v} \times \mathbf{B} \quad (2)$$

$$= \omega a B \sin \theta \hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} \quad (3)$$

The vector $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}}$ points horizontally outward. If we now want to find the emf induced by the field between the north pole (at $\theta = 0$) and the equator (at $\theta = \frac{\pi}{2}$), we can do the integral above over a path along a line of longitude between those two angles. Along this path

$$d\boldsymbol{\ell} = a(d\theta) \hat{\boldsymbol{\theta}} \quad (4)$$

The angle between $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}}$ and $\hat{\boldsymbol{\theta}}$ is also θ so

$$\mathcal{E} = \oint \mathbf{f} \cdot d\boldsymbol{\ell} \quad (5)$$

$$= \omega a^2 B \int_0^{\pi/2} \sin \theta (\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}}) \cdot \hat{\boldsymbol{\theta}} \quad (6)$$

$$= \omega a^2 B \int_0^{\pi/2} \sin \theta \cos \theta d\theta \quad (7)$$

$$= \frac{1}{2} \omega a^2 B \int_0^{\pi/2} \sin 2\theta d\theta \quad (8)$$

$$= \frac{1}{2} \omega a^2 B \quad (9)$$

Note that this is the same emf as that produced in a flat, circular disk of radius a rotating at angular speed ω (Griffiths, example 7.4) so the sphere problem effectively reduces to the disk problem by projecting the sphere onto its equatorial plane.