CHANGING A PARTICLE'S SPEED IN A CYCLOTRON

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.48.

We can now revisit the problem of a charged particle in a cyclotron field. Suppose we start with a charged particle of mass m and charge q at rest a distance R from the axis of the cyclotron, and we want to increase the speed of the particle in its circular orbit while keeping it at the same radius (such a device is called a *betatron*). We can do this by varying the magnetic field $\mathbf{B}(t)$ such that the cyclotron relation is always satisfied:

$$qvB = \frac{mv^2}{R} \tag{1}$$

$$B = \frac{mv}{qR} \tag{2}$$

Taking the time derivative, we get

$$\dot{B} = \frac{m\dot{v}}{qR} \tag{3}$$

The changing magnetic field induces a circumferential electric field $\mathbf{E}$, and since this field is parallel to the particle's direction of motion it will act as the force that accelerates the particle. From Newton's law, $F = m\dot{v} = qE$, so

$$\dot{B} = \frac{E}{R} \tag{4}$$

If we assume the cyclotron has cylindrical symmetry, then we can integrate this equation along the particle's orbit, along which both fields are constant in magnitude. That is

$$\oint \dot{B}_c d\ell = \oint \frac{E}{R} d\ell \tag{5}$$

$$\oint E d\ell = R \oint \dot{B}_c d\ell \tag{6}$$

$$= 2\pi R^2 \dot{B}_c \tag{7}$$

This equation applies on the circumference of the orbit only (not in the interior of the orbit), so we've added a suffix c to B_c to emphasize this point.

From Faraday's law in integral form, we have also that the integral of the electric field is given by the change in flux:

$$\oint E d\ell = -\frac{d\Phi}{dt} \tag{8}$$

$$= -\int \dot{\mathbf{B}}\cdot d\mathbf{a} \tag{9}$$

where now we are integrating over all points within the orbit.

If we start with the particle at rest in zero field, then we have

$$2\pi R^2 \dot{B}_c = -\int \dot{\mathbf{B}}\cdot d\mathbf{a} = \pi R^2 \dot{\bar{B}} \tag{10}$$

where $\bar{B}$ is the average field across the orbit.

A word about the signs here. Suppose the magnetic field points in the $-z$ direction (as shown in Fig. 7.52 in Griffiths), and we take the area vector to point in the $+z$ direction. Further, if the magnetic field is increasing in magnitude, then $\dot{\mathbf{B}}$ points towards $-z$ as well. Thus $\dot{\mathbf{B}}\cdot d\mathbf{a} < 0$, giving the sign shown.

We can integrate both sides to some time t to get

$$\dot{B}_c(t) = \frac{1}{2}\bar{B}(t) \tag{11}$$

Thus we can speed up (or slow down, by decreasing the field) the particle by keeping the average field equal to twice the field at the radius of the orbit.