A BIZARRE EXAMPLE OF FARADAY'S LAW

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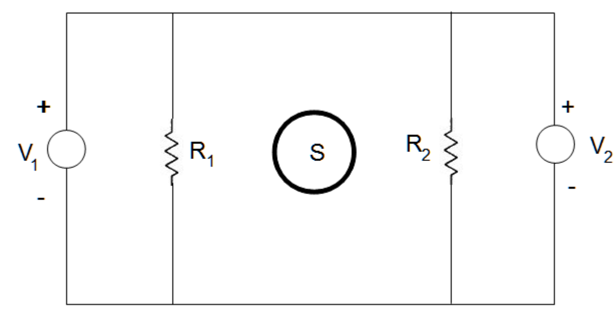
References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.50.

Romer, Robert. H. (1982), Am. J. Phys. **50**, 1089.

This is a rather bizarre illustration of Faraday's law. Suppose we set up the circuit as shown in the figure:



The circle marked S is a solenoid perpendicular to the plane of the screen, and V_1 and V_2 are two ideal voltmeters (in the sense that they have infinite resistance and thus draw no current). The current through the solenoid is varied at a constant rate such that the magnetic flux in the interior of the solenoid varies according to $\Phi(t)=\alpha t$, where α is a constant. According to Faraday's law, the changing flux induces an electric field such that

$$\oint\mathbf{E}\cdot d\ell=-\alpha \tag{0.1}$$

for any path enclosing the solenoid. If the magnetic field points into the screen, then by the right hand rule, the current through the solenoid runs clockwise around the solenoid. By Lenz's law, the induced electric field opposes the flux change, so it runs counterclockwise around the loop containing the two resistors, and the current is given by

$$\alpha=I(R_1+R_2) \tag{0.2}$$

The voltage drop across R_1 is therefore

$$V_1 = IR_1 \tag{0.3}$$

$$= \frac{\alpha R_2}{R_1 + R_2} \tag{0.4}$$

That is, the voltage drops by this amount when we travel *up* the branch of the circuit containing R_1. Similarly, if we travel *down* the branch containing R_2, the voltage drop is

$$V_2 = -\frac{\alpha R_2}{R_1 + R_2} \tag{0.5}$$

where the minus sign is because both voltmeters are measuring the voltage for a path travelling upwards.

This result seems bizarre because both meters are essentially connected to the same points in the circuit. In his paper, Romer does an experiment in which he actually measures the two voltages and shows that they do actually behave this way. The key point seems to be that what we are measuring is the actual integral $\int \mathbf{E} \cdot d\ell$ along a specific path, so if we put both voltmeters on the same side of the solenoid, then they would read the same.