SUPERCONDUCTING LOOP IN A MAGNETIC FIELD

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 7.51.

When we introduced the idea of motional emf, we used an example of a circuit containing a resistance R. What happens if the circuit contains no resistance (as with a superconductor)? In this case, we need to consider the self-inductance of the circuit. As a simple example, suppose we have a rectangular loop of wire with dimensions of h and ℓ (height and length). The loop is placed in a magnetic field that is perpendicular to the plane of the loop, and which covers a distance $x<\ell$ along the length. If the loop is free to move parallel to its length, what motion does it follow, assuming that it is given an initial velocity v_0?

The motion of the loop within the field causes a current to flow due to the Lorentz force $q\mathbf{v}\times\mathbf{B}$. This force is perependicular to the wire along the top and bottom edges of the loop, so it is only the end of the loop within the field that experiences the force, since here the Lorentz force is parallel to the wire. Suppose the current is I. This current flows up the end of the loop, which means that an additional component of the Lorentz force acts. If the field points into the page and the current is flowing upwards, then this force is to the left. If the speed of the charges along the wire is u, the total force on the end of the loop is λhuB, where λ is the linear charge density. However, the current is the amount of charge that flows past a point in unit time, so the total charge in that end of the loop is $I=\lambda u$ so the force is IhB.

Now, the self-inductance of a loop relates the magnetic flux through the loop to the current:

$$\frac{d\Phi}{dt} = L\dot{I} \tag{1}$$

The rate of change of flux is

$$\frac{d\Phi}{dt}=hB\dot{x} \tag{2}$$

so

$$\dot{I} = \frac{Bh}{L}\dot{x} \tag{3}$$

$$I = \frac{Bh}{L}(x - x_0) \tag{4}$$

where x_0 is the position at time $t = 0$. Thus $I < 0$ (upwards) as the loop's edge nears the edge of the field, and $I > 0$ (downwards) when the edge is far from the edge of the field. If we define the positive direction as the direction of increasing x, this means that positive is to the *left* here. This means that a *negative* current gives a *positive* force, so

$$F = -IhB \tag{5}$$

$$m\ddot{x} = -\frac{B^2h^2}{L}(x - x_0) \tag{6}$$

This has the general solution

$$x(t) = A\sin(\omega t + \phi_0) + x_0 \tag{7}$$

with the frequency given by

$$\omega = \frac{Bh}{\sqrt{mL}} \tag{8}$$