TRANSMISSION LINES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problems 7.58.

In this problem, we've got a transmission line consisting of two thin ribbons (each of width w) separated by an insulating vacuum space of thickness $h \ll w$. We've looked at a similar problem before, so we'll start from there. If we assume that the magnetic field is confined to the region between the ribbons (in reality there will be some fringe effects, but if the ribbons are very close together, these will be small), then the field per unit length is

$$B = \mu_0 \sigma v \tag{1}$$

where σ is the surface charge density and v is the speed of the current I, so that $I = \sigma v w$ and

$$B = \frac{\mu_0 I}{w} \tag{2}$$

The energy in a magnetic field is

$$W_B = \frac{1}{2} L I^2 = \frac{1}{2\mu_0} \int B^2 d^3\mathbf{r} \tag{3}$$

The volume between a unit length of ribbons is hw and we can take the field to be constant in this volume, so we can get the inductance per unit length:

$$\frac{1}{2} L I^2 = \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{w} \right)^2 hw \tag{4}$$

$$L = \mu_0 \frac{h}{w} \tag{5}$$

The capacitance of a unit length can be found by treating the cable as a parallel plate capacitor:

$$C = \frac{\epsilon_0 w}{h} \tag{6}$$

The product is thus:

$$LC=\mu_0\varepsilon_0=\frac{1}{c^2} \tag{7}$$

where c is the speed of light.

If the ribbons are separated by a material with permittivity ε and permeability μ then we get (see Griffiths, Example 4.6):

$$C=\frac{\varepsilon}{\varepsilon_0}\frac{\varepsilon_0 w}{h}=\frac{\varepsilon w}{h} \tag{8}$$

For the inductance, we can start from the definition of inductance, which is $\Phi=LI$. The flux is $\int\mathbf{B}\cdot d\mathbf{a}$ and within matter, $\mathbf{B}=\mu\mathbf{B}_{vac}/\mu_0$ so L must scale the same way:

$$L=\frac{\mu}{\mu_0}\mu_0\frac{h}{w}=\mu\frac{h}{w} \tag{9}$$

and the product is

$$LC=\mu\varepsilon \tag{10}$$

If the velocity is taken as $v=1/\sqrt{LC}$ we get

$$v=\frac{1}{\sqrt{\mu\varepsilon}} \tag{11}$$

PINGBACKS

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