

## DUALITY TRANSFORMATION FOR MAGNETIC AND ELECTRIC CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 7, Post 60.

Continuing our excursion in the fantasy world where magnetic monopoles exist, we can explore the *duality transformations*, which describe a rotation in an E-B space, as follows:

$$(0.1) \quad \mathbf{E}' = \mathbf{E} \cos \alpha + c\mathbf{B} \sin \alpha$$

$$(0.2) \quad c\mathbf{B}' = c\mathbf{B} \cos \alpha - \mathbf{E} \sin \alpha$$

$$(0.3) \quad cq'_e = cq_e \cos \alpha + q_m \sin \alpha$$

$$(0.4) \quad q'_m = q_m \cos \alpha - cq_e \sin \alpha$$

where  $\alpha$  is an arbitrary angle and  $c = 1/\sqrt{\mu_0\epsilon_0}$ . Currents and charge densities transform the same way as the corresponding point charges.

It's a straightforward, though tedious, exercise to verify that Maxwell's equations (including magnetic charge) are invariant under these transformations. The two equations that are different are:

$$(0.5) \quad \nabla \cdot \mathbf{B} = \mu_0 \rho_m$$

$$(0.6) \quad \nabla \times \mathbf{E} = -\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}$$

A few sample calculations should show how it goes. We'll do one involving a divergence:

$$(0.7) \quad c\nabla \cdot \mathbf{B}' = c\nabla \cdot \mathbf{B} \cos \alpha - \nabla \cdot \mathbf{E} \sin \alpha$$

$$(0.8) \quad = c\mu_0 \rho_m \cos \alpha - \frac{\rho_e}{\epsilon_0} \sin \alpha$$

$$(0.9) \quad = c\mu_0 (\rho_m \cos \alpha - c\rho_e \sin \alpha)$$

$$(0.10) \quad = c\mu_0 \rho'_m$$

And another involving a curl:

$$(0.11) \quad \nabla \times \mathbf{E}' = \nabla \times \mathbf{E} \cos \alpha + c \nabla \times \mathbf{B} \sin \alpha$$

$$(0.12) \quad = \left( -\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t} \right) \cos \alpha + c \left( \mu_0 \mathbf{J}_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \sin \alpha$$

$$(0.13) \quad = -\mu_0 (\mathbf{J}_m \cos \alpha - c \mathbf{J}_e \sin \alpha) - \frac{\partial \mathbf{B}}{\partial t} \cos \alpha + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \sin \alpha$$

$$(0.14) \quad = -\mu_0 \mathbf{J}'_m - \frac{\partial \mathbf{B}'}{\partial t}$$

The other two equations transform similarly.

If we take the force law for electric and magnetic charges to be

$$(0.15) \quad \mathbf{F} = q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m \left( \mathbf{B} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right)$$

then the transformed law is

$$(0.16) \quad \mathbf{F}' = q'_e (\mathbf{E}' + \mathbf{v} \times \mathbf{B}') + q'_m \left( \mathbf{B}' - \frac{1}{c^2} \mathbf{v} \times \mathbf{E}' \right)$$

$$(0.17) \quad = \left( q_e \cos \alpha + \frac{q_m}{c} \sin \alpha \right) \times$$

$$(0.18) \quad \left[ \mathbf{E} \cos \alpha + c \mathbf{B} \sin \alpha + \mathbf{v} \times \left( \mathbf{B} \cos \alpha - \frac{1}{c} \mathbf{E} \sin \alpha \right) \right] +$$

$$(0.19) \quad (q_m \cos \alpha - c q_e \sin \alpha) \times$$

$$(0.20) \quad \left[ \mathbf{B} \cos \alpha - \frac{1}{c} \mathbf{E} \sin \alpha - \frac{1}{c^2} \mathbf{v} \times (\mathbf{E} \cos \alpha + c \mathbf{B} \sin \alpha) \right]$$

$$(0.21) \quad = q_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m \left( \mathbf{B} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right)$$

$$(0.22) \quad = \mathbf{F}$$

To get the penultimate line it is just a matter of multiplying out the first expression and using  $\cos^2 \alpha + \sin^2 \alpha = 1$ , and cancelling off terms.

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