WAVE EQUATION: WAVE SPEED AND STANDING WAVES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.2.

Any function of form $f=f(z\pm vt)$ (for a constant v) is a solution of the wave equation

$$\frac{\partial^2 f}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \tag{1}$$

Once we specify f, we can plot it as a function of z for a fixed value of t, say t_1. This shows a snapshot of the wave at a particular time. If we then plot f as a function of z at some later time t_2, the wave has the same shape but is shifted along the z axis. A point at which f has the value $f_1\equiv f(z_1-vt_1)$ will have moved to a position z_2 where $z_2-vt_2=z_1-vt_1$. In other words, that particular point on the wave moves from z_1 to z_2 in a time t_2-t_1, and must be moving with speed

$$\frac{z_2-z_1}{t_2-t_1}=\frac{v(t_2-t_1)}{t_2-t_1}=v \tag{2}$$

Thus v is the speed of the wave. If $z_2>z_1$, then $v>0$ and the wave moves towards increasing z. If $z_2<z_1$, then $v<0$ and the wave moves towards decreasing z.

Since the wave equation is linear, any linear combination of solutions is also a solution, so we can add together waves with equal and opposite speeds. For example, if we have two sine waves with equal and opposite speeds, we can add them together to get

$$\begin{aligned} f(z,t)=\sin(k(z-vt))+\sin(k(z+vt)) &= \sin kz\cos kvt-\cos kz\sin kvt+ \\ &\quad \sin kz\cos kvt+\cos kz\sin kvt \qquad (3)\\ &= 2\sin kz\cos kvt \qquad (4)\end{aligned}$$

where k is a constant.

The latter form is not a function of form $f(z\pm vt)$ but as it's a sum of two functions of this type, it is still a solution of the wave equation. In fact, it represents a standing wave, that is, a wave which doesn't travel along the

z axis, but rather oscillates in place. We can see this since for all points $z_n = n\pi/k$ where n is an integer, $f(z_n,t) = 0$ for all times t. For all times $t_n = (2n+1)\pi/2kv$, $f(z,t_n) = 0$ for all positions, so the wave becomes a flat line. In between these times, the wave oscillates between its maximum and minimum heights of $\pm 2\sin kz$.

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