ELECTROMAGNETIC WAVES: ENERGY, MOMENTUM AND LIGHT PRESSURE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.10.

We've seen that Maxwell's equations in a vacuum predict that an electromagnetic field can propagate as a wave with the speed of light. We've also seen that any region where both an electric and magnetic field exists results in energy flowing through the region. The rate per unit area at which energy crosses a surface is given by the Poynting vector

$$\mathbf{S}=\frac{1}{\mu_{0}}\mathbf{E}\times\mathbf{B} \tag{0.1}$$

In an electromagnetic wave, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other, and also perpendicular to the direction $\mathbf{k}$ of propagation of the wave. Thus their cross product is parallel to $\mathbf{k}$, indicating that an electromagnetic wave carries energy.

Electromagnetic fields also carry momentum, with the momentum density given by

$$\mathfrak{p}=\varepsilon_{0}\mu_{0}\mathbf{S}=\frac{1}{c^{2}}\mathbf{S} \tag{0.2}$$

For a monochromatic plane wave travelling in the z direction

$$\begin{aligned}
\mathbf{E} &= E_{0}\cos\left(kz-\omega t+\delta\right)\hat{\mathbf{x}} && (0.3)\\
\mathbf{B} &= \frac{E_{0}}{c}\cos\left(kz-\omega t+\delta\right)\hat{\mathbf{y}} && (0.4)\\
\mathbf{S} &= \frac{E_{0}^{2}}{\mu_{0}c}\cos^{2}\left(kz-\omega t+\delta\right)\hat{\mathbf{z}} && (0.5)\\
&= E_{0}^{2}c\varepsilon_{0}\cos^{2}\left(kz-\omega t+\delta\right)\hat{\mathbf{z}} && (0.6)
\end{aligned}$$

The average of cos or sin squared over a complete cycle is $\frac{1}{2}$, so the averages of energy and momentum density are

$$\langle \mathbf{S} \rangle = \frac{1}{2} E_0^2 c \varepsilon_0 \hat{\mathbf{z}} \tag{0.7}$$

$$\langle \mathfrak{p} \rangle = \frac{1}{2c} E_0^2 \varepsilon_0 \hat{\mathbf{z}} \tag{0.8}$$

The magnitude of $\langle \mathbf{S} \rangle$ has the units of energy per unit area per unit time, or power per unit area, and is known as the *intensity* of the radiation:

$$I \equiv \langle S \rangle = \frac{1}{2} E_0^2 c \varepsilon_0 \tag{0.9}$$

$$\langle \mathfrak{p} \rangle = \frac{1}{2c} E_0^2 \varepsilon_0 = \frac{I}{c^2} \tag{0.10}$$

If an electromagnetic wave hits a surface, its momentum is either absorbed or reflected (or bits of both), so EM radiation actually exerts a pressure on a surface. Pressure is force per unit area, which is momentum transferred per unit time per unit area. The quantity $\langle \mathfrak{p} \rangle$ is the momentum density in the wave, so the amount of momentum that falls upon an area A in time Δt is the volume of the wave that falls on the area times $\langle \mathfrak{p} \rangle$. The wave is travelling at speed c so this volume is $Ac\Delta t$ and the momentum that falls on the area is

$$\Delta p = \langle \mathfrak{p} \rangle Ac\Delta t \tag{0.11}$$

The average pressure is the average force per unit area, which in turn is the momentum received by the area per unit area per unit time, so

$$P = \frac{\Delta p}{A\Delta t} = \langle \mathfrak{p} \rangle c = \frac{1}{2} E_0^2 \varepsilon_0 = \frac{I}{c} \tag{0.12}$$

This assumes a perfect absorber, so the radiation is just absorbed and not reflected. A perfect reflector would experience a pressure twice as large, since the momentum in the fields is not just stopped, it is reversed.

To get an idea of how big this pressure is, suppose we're outside on a sunny day at noon, when the solar radiation is at its maximum. At my latitude (Scotland) this intensity never gets very far above 1000 Watts m^{-2} (if you're interested in my weather data see here) but we'll use Griffiths's value of 1300 Watts m^{-2}. If this sunlight strikes a perfect absorber then

$$P = \frac{1300}{3 \times 10^8} = 4.33 \times 10^{-6}\ \mathrm{N\ m}^{-2} \tag{0.13}$$

For a perfect reflector, the pressure is just twice this, or 8.66×10^{-6} N m^{-2}.

For comparison, standard atmospheric pressure is 101325 N m^{-2} so it's no surprise that the pressure exerted by sunlight isn't noticeable.

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