ELECTROMAGNETIC WAVES IN MATTER: REFLECTION AND TRANSMISSION COEFFICIENTS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.13.

Continuing with our study of electromagentic waves in matter, we'll carry on with the system of an incident wave travelling in the $+z$ direction (so $\hat{\mathbf{k}} = \hat{\mathbf{z}}$) and polarized in the x direction (so $\hat{\mathbf{n}} = \hat{\mathbf{x}}$). Suppose the boundary is the xy plane, with medium 1 on the left ($z < 0$) and medium 2 on the right ($z > 0$). Then the reflected and transmitted (complex) amplitudes are

$$\tilde{E}_{0_R} = \pm\frac{1-\beta}{1+\beta}\tilde{E}_{0_I} \tag{1}$$

$$\tilde{E}_{0_T} = \pm\frac{2}{1+\beta}\tilde{E}_{0_I} \tag{2}$$

where $\tilde{E}_{0_I}$ is the incident amplitude and

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} \tag{3}$$

with v_i the speed of the wave in medium i and $n_i = c/v_i$ the index of refraction.

The intensity of a wave in a vacuum is defined as the mean (over time) of the magnitude of the Poynting vector:

$$I \equiv \langle S \rangle = \frac{1}{2}E_0^2 c\epsilon_0 \tag{4}$$

If we follow through the derivation of I for a wave in matter, we see that the only difference is that c is replaced by v and ϵ_0 by ϵ, so the intensity becomes

$$I = \frac{1}{2}\epsilon v E_0^2 \tag{5}$$

The reflection coefficient R is the ratio of reflected to incident intensity:

$$R = \frac{\frac{1}{2}\epsilon_1 v_1 E_{0_R}^2}{\frac{1}{2}\epsilon_1 v_1 E_{0_I}^2} \tag{6}$$

$$= \left(\frac{1-\beta}{1+\beta}\right)^2 \tag{7}$$

The transmission coefficient T is the ratio of transmitted to incident intensity:

$$T = \frac{\frac{1}{2}\epsilon_2 v_2 E_{0_T}^2}{\frac{1}{2}\epsilon_1 v_1 E_{0_I}^2} \tag{8}$$

$$= \frac{4\epsilon_2 v_2}{\epsilon_1 v_1 (1+\beta)^2} \tag{9}$$

$$= \frac{4\beta}{(1+\beta)^2} \tag{10}$$

where in the last line we used

$$v_i = \frac{1}{\sqrt{\epsilon_i \mu_i}} \tag{11}$$

$$\epsilon_i = \frac{1}{\mu_i v_i^2} \tag{12}$$

We can see that $R+T=1$ which is just an expression of the conservation of energy. The larger n_2 is relative to n_1, the larger is β which means that $R \to 1$ and $T \to 0$.

The theory here is incomplete, as in practice the index of refraction depends not only on the material but also on the wavelength of radiation. This is largely a quantum phenomenon as it depends on the distances between the atoms in the refracting medium, whereas the classical theory assumes the medium is continuous.

PINGBACKS

Pingback: Fresnel equations for perpendicular polarization
Pingback: Reflection at a conducting surface: the physics of mirrors
Pingback: Transmission coefficient for a wave passing through 3 media