ELECTROMAGNETIC WAVES IN MATTER: NORMAL REFLECTION AND TRANSMISSION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.14.

To see how electromagnetic waves propagate in matter, we can start with Maxwell's equations in matter:

$$\nabla\cdot\mathbf{D} = \rho_f \tag{0.1}$$
$$\nabla\cdot\mathbf{B} = 0 \tag{0.2}$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{0.3}$$
$$\nabla\times\mathbf{H} = \mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t} \tag{0.4}$$

In the medium is linear, homogeneous and contains no free charge or current, these equations reduce to

$$\nabla\cdot\mathbf{E} = 0 \tag{0.5}$$
$$\nabla\cdot\mathbf{B} = 0 \tag{0.6}$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{0.7}$$
$$\nabla\times\mathbf{B} = \mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \tag{0.8}$$

These equations are identical to those for a vacuum, except that the permeability ε_0 and permittivity μ_0 of free space have been replaced by their corresponding values ε and μ in the medium. Thus electromagnetic waves propagate the same way in a medium, so we can write them as

$$\tilde{\mathbf{E}} = \tilde{E}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{n}} \tag{0.9}$$
$$\tilde{\mathbf{B}} = \tilde{B}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{k}}\times\hat{\mathbf{n}} \tag{0.10}$$
$$= \frac{1}{v}\hat{\mathbf{k}}\times\tilde{\mathbf{E}} \tag{0.11}$$

where

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \tag{0.12}$$

is the speed of the wave in the medium. Since the permittivities and permeabilities of materials are almost always greater than those for free space, the speed $v < c$ in almost all cases, so light travels more slowly through a medium than through a vacuum. The ratio of the speeds is the *index of refraction*:

$$n \equiv \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \tag{0.13}$$

We can apply the boundary conditions at the interface between two media to find out how light behaves when passing from one medium into another.

$$\begin{aligned}
\varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} &= 0 && (0.14)\\
B_1^{\perp} - B_2^{\perp} &= 0 && (0.15)\\
\mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} &= 0 && (0.16)\\
\frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} &= 0 && (0.17)
\end{aligned}$$

We'll start with an incident wave travelling in the $+z$ direction (so $\hat{\mathbf{k}} = \hat{\mathbf{z}}$) and polarized in the x direction (so $\hat{\mathbf{n}} = \hat{\mathbf{x}}$) and suppose the boundary is the xy plane, with medium 1 on the left ($z < 0$) and medium 2 on the right ($z > 0$). Then the incident wave is

$$\begin{aligned}
\tilde{\mathbf{E}}_I &= \tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\hat{\mathbf{x}} && (0.18)\\
\tilde{\mathbf{B}}_I &= \tilde{B}_{0_I} e^{i(k_1 z - \omega t)}\hat{\mathbf{z}} \times \hat{\mathbf{x}} && (0.19)\\
&= \frac{1}{v_1}\tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\hat{\mathbf{y}} && (0.20)
\end{aligned}$$

We'll have a reflected wave ($\hat{\mathbf{k}} = -\hat{\mathbf{z}}$) and a transmitted wave ($\hat{\mathbf{k}} = \hat{\mathbf{z}}$), so

$$\begin{aligned}
\tilde{\mathbf{E}}_R &= \tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\left(\cos\theta_R\hat{\mathbf{x}} + \sin\theta_R\hat{\mathbf{y}}\right) && (0.21)\\
\tilde{\mathbf{B}}_R &= -\tilde{B}_{0_R} e^{i(-k_1 z - \omega t)}\hat{\mathbf{z}} \times \left(\cos\theta_R\hat{\mathbf{x}} + \sin\theta_R\hat{\mathbf{y}}\right) && (0.22)\\
&= \frac{1}{v_1}\tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\left(\sin\theta_R\hat{\mathbf{x}} - \cos\theta_R\hat{\mathbf{y}}\right) && (0.23)
\end{aligned}$$

$$\tilde{\mathbf{E}}_T = \tilde{E}_{0_T} e^{i(k_2 z - \omega t)} (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{0.24}$$

$$\tilde{\mathbf{B}}_T = \tilde{B}_{0_T} e^{i(k_2 z - \omega t)} \hat{\mathbf{z}} \times (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{0.25}$$

$$= \frac{1}{v_2} \tilde{E}_{0_T} e^{i(k_2 z - \omega t)} (-\sin\theta_T \hat{\mathbf{x}} + \cos\theta_T \hat{\mathbf{y}}) \tag{0.26}$$

where θ_R and θ_T are the angles of polarization for the reflected and transmitted waves.

Since there are no components of the fields perpendicular to the boundary at $z = 0$, 0.14 and 0.15 tell us nothing. Applying 0.16 to the x and y components, we have

$$\tilde{E}_{0_I} \hat{\mathbf{x}} + \tilde{E}_{0_R} (\cos\theta_R \hat{\mathbf{x}} + \sin\theta_R \hat{\mathbf{y}}) = \tilde{E}_{0_T} (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{0.27}$$

$$\tilde{E}_{0_I} + \tilde{E}_{0_R} \cos\theta_R = \tilde{E}_{0_T} \cos\theta_T \tag{0.28}$$

$$\tilde{E}_{0_R} \sin\theta_R = \tilde{E}_{0_T} \sin\theta_T \tag{0.29}$$

From 0.17 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0_R} \sin\theta_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \sin\theta_T \tag{0.30}$$

$$\frac{1}{\mu_1 v_1} \left(\tilde{E}_{0_I} - \tilde{E}_{0_R} \cos\theta_R \right) = \frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \cos\theta_T \tag{0.31}$$

Substituting from 0.29 into 0.30 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0_T} \sin\theta_T = -\frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \sin\theta_T \tag{0.32}$$

The only way this equation can be satisfied is if $\sin\theta_T = 0$ so from 0.29 we conclude that $\sin\theta_R = 0$ as well. Thus θ_T and θ_R are either 0 or π, so the cosines are ± 1. Thus the plane of polarization is the same for the reflected and transmitted waves as for the incident wave.

If we define

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} \tag{0.33}$$

we can write 0.31 as

$$\tilde{E}_{0_I} - \tilde{E}_{0_R} \cos\theta_R = \beta \tilde{E}_{0_T} \cos\theta_T \tag{0.34}$$

Adding this to 0.28 we get

$$\tilde{E}_{0_I} = \frac{1+\beta}{2}\tilde{E}_{0_T}\cos\theta_T \tag{0.35}$$

$$\tilde{E}_{0_T} = \pm\frac{2}{1+\beta}\tilde{E}_{0_I} \tag{0.36}$$

Subtracting 0.34 from 0.28 we get

$$\tilde{E}_{0_R} = \pm\frac{1-\beta}{2}\tilde{E}_{0_T} = \pm\frac{1-\beta}{1+\beta}\tilde{E}_{0_I} \tag{0.37}$$

Thus the reflected and transmitted waves are either in phase or half a cycle out of phase with the incident wave.

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