

## ELECTROMAGNETIC WAVES IN MATTER: NORMAL REFLECTION AND TRANSMISSION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.14.

To see how electromagnetic waves propagate in matter, we can start with Maxwell's equations in matter:

$$\nabla \cdot \mathbf{D} = \rho_f \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

In the medium is linear, homogeneous and contains no free charge or current, these equations reduce to

$$\nabla \cdot \mathbf{E} = 0 \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (7)$$

$$\nabla \times \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (8)$$

These equations are identical to those for a vacuum, except that the permeability  $\epsilon_0$  and permittivity  $\mu_0$  of free space have been replaced by their corresponding values  $\epsilon$  and  $\mu$  in the medium. Thus electromagnetic waves propagate the same way in a medium, so we can write them as

$$\tilde{\mathbf{E}} = \tilde{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{\mathbf{n}} \quad (9)$$

$$\tilde{\mathbf{B}} = \tilde{B}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{\mathbf{k}} \times \hat{\mathbf{n}} \quad (10)$$

$$= \frac{1}{v} \hat{\mathbf{k}} \times \tilde{\mathbf{E}} \quad (11)$$

where

$$v = \frac{1}{\sqrt{\mu\epsilon}} \quad (12)$$

is the speed of the wave in the medium. Since the permittivities and permeabilities of materials are almost always greater than those for free space, the speed  $v < c$  in almost all cases, so light travels more slowly through a medium than through a vacuum. The ratio of the speeds is the *index of refraction*:

$$n \equiv \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} \quad (13)$$

We can apply the boundary conditions at the interface between two media to find out how light behaves when passing from one medium into another.

$$\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = 0 \quad (14)$$

$$B_1^\perp - B_2^\perp = 0 \quad (15)$$

$$\mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel = 0 \quad (16)$$

$$\frac{1}{\mu_1} \mathbf{B}_1^\parallel - \frac{1}{\mu_2} \mathbf{B}_2^\parallel = 0 \quad (17)$$

We'll start with an incident wave travelling in the  $+z$  direction (so  $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ ) and polarized in the  $x$  direction (so  $\hat{\mathbf{n}} = \hat{\mathbf{x}}$ ) and suppose the boundary is the  $xy$  plane, with medium 1 on the left ( $z < 0$ ) and medium 2 on the right ( $z > 0$ ). Then the incident wave is

$$\tilde{\mathbf{E}}_I = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \quad (18)$$

$$\tilde{\mathbf{B}}_I = \tilde{B}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{z}} \times \hat{\mathbf{x}} \quad (19)$$

$$= \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \quad (20)$$

We'll have a reflected wave ( $\hat{\mathbf{k}} = -\hat{\mathbf{z}}$ ) and a transmitted wave ( $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ ), so

$$\tilde{\mathbf{E}}_R = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}}) \quad (21)$$

$$\tilde{\mathbf{B}}_R = -\tilde{B}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{z}} \times (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}}) \quad (22)$$

$$= \frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} (\sin \theta_R \hat{\mathbf{x}} - \cos \theta_R \hat{\mathbf{y}}) \quad (23)$$

$$\tilde{\mathbf{E}}_T = \tilde{E}_{0T} e^{i(k_2 z - \omega t)} (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}}) \quad (24)$$

$$\tilde{\mathbf{B}}_T = \tilde{B}_{0T} e^{i(k_2 z - \omega t)} \hat{\mathbf{z}} \times (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}}) \quad (25)$$

$$= \frac{1}{v_2} \tilde{E}_{0T} e^{i(k_2 z - \omega t)} (-\sin \theta_T \hat{\mathbf{x}} + \cos \theta_T \hat{\mathbf{y}}) \quad (26)$$

where  $\theta_R$  and  $\theta_T$  are the angles of polarization for the reflected and transmitted waves.

Since there are no components of the fields perpendicular to the boundary at  $z = 0$ , 14 and 15 tell us nothing. Applying 16 to the  $x$  and  $y$  components, we have

$$\tilde{E}_{0I} \hat{\mathbf{x}} + \tilde{E}_{0R} (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}}) = \tilde{E}_{0T} (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}}) \quad (27)$$

$$\tilde{E}_{0I} + \tilde{E}_{0R} \cos \theta_R = \tilde{E}_{0T} \cos \theta_T \quad (28)$$

$$\tilde{E}_{0R} \sin \theta_R = \tilde{E}_{0T} \sin \theta_T \quad (29)$$

From 17 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0R} \sin \theta_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin \theta_T \quad (30)$$

$$\frac{1}{\mu_1 v_1} (\tilde{E}_{0I} - \tilde{E}_{0R} \cos \theta_R) = \frac{1}{\mu_2 v_2} \tilde{E}_{0T} \cos \theta_T \quad (31)$$

Substituting from 29 into 30 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0T} \sin \theta_T = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin \theta_T \quad (32)$$

The only way this equation can be satisfied is if  $\sin \theta_T = 0$  so from 29 we conclude that  $\sin \theta_R = 0$  as well. Thus  $\theta_T$  and  $\theta_R$  are either 0 or  $\pi$ , so the cosines are  $\pm 1$ . Thus the plane of polarization is the same for the reflected and transmitted waves as for the incident wave.

If we define

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} \quad (33)$$

we can write 31 as

$$\tilde{E}_{0I} - \tilde{E}_{0R} \cos \theta_R = \beta \tilde{E}_{0T} \cos \theta_T \quad (34)$$

Adding this to 28 we get

$$\tilde{E}_{0I} = \frac{1+\beta}{2} \tilde{E}_{0T} \cos \theta_T \quad (35)$$

$$\tilde{E}_{0T} = \pm \frac{2}{1+\beta} \tilde{E}_{0I} \quad (36)$$

Subtracting 34 from 28 we get

$$\tilde{E}_{0R} = \pm \frac{1-\beta}{2} \tilde{E}_{0T} = \pm \frac{1-\beta}{1+\beta} \tilde{E}_{0I} \quad (37)$$

Thus the reflected and transmitted waves are either in phase or half a cycle out of phase with the incident wave.

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