

ELECTROMAGNETIC WAVES IN MATTER: NORMAL REFLECTION AND TRANSMISSION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.14.

To see how electromagnetic waves propagate in matter, we can start with Maxwell's equations in matter:

$$\begin{aligned}(1) \quad & \nabla \cdot \mathbf{D} = \rho_f \\(2) \quad & \nabla \cdot \mathbf{B} = 0 \\(3) \quad & \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\(4) \quad & \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

In the medium is linear, homogeneous and contains no free charge or current, these equations reduce to

$$\begin{aligned}(5) \quad & \nabla \cdot \mathbf{E} = 0 \\(6) \quad & \nabla \cdot \mathbf{B} = 0 \\(7) \quad & \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\(8) \quad & \nabla \times \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

These equations are identical to those for a vacuum, except that the permeability ϵ_0 and permittivity μ_0 of free space have been replaced by their corresponding values ϵ and μ in the medium. Thus electromagnetic waves propagate the same way in a medium, so we can write them as

$$\begin{aligned}(9) \quad & \tilde{\mathbf{E}} = \tilde{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{\mathbf{n}} \\(10) \quad & \tilde{\mathbf{B}} = \tilde{B}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{\mathbf{k}} \times \hat{\mathbf{n}} \\(11) \quad & = \frac{1}{v} \hat{\mathbf{k}} \times \tilde{\mathbf{E}}\end{aligned}$$

where

$$(12) \quad v = \frac{1}{\sqrt{\mu\epsilon}}$$

is the speed of the wave in the medium. Since the permittivities and permeabilities of materials are almost always greater than those for free space, the speed $v < c$ in almost all cases, so light travels more slowly through a medium than through a vacuum. The ratio of the speeds is the *index of refraction*:

$$(13) \quad n \equiv \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

We can apply the boundary conditions at the interface between two media to find out how light behaves when passing from one medium into another.

$$(14) \quad \epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = 0$$

$$(15) \quad B_1^\perp - B_2^\perp = 0$$

$$(16) \quad \mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel = 0$$

$$(17) \quad \frac{1}{\mu_1} \mathbf{B}_1^\parallel - \frac{1}{\mu_2} \mathbf{B}_2^\parallel = 0$$

We'll start with an incident wave travelling in the $+z$ direction (so $\hat{\mathbf{k}} = \hat{\mathbf{z}}$) and polarized in the x direction (so $\hat{\mathbf{n}} = \hat{\mathbf{x}}$) and suppose the boundary is the xy plane, with medium 1 on the left ($z < 0$) and medium 2 on the right ($z > 0$). Then the incident wave is

$$(18) \quad \tilde{\mathbf{E}}_I = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}}$$

$$(19) \quad \tilde{\mathbf{B}}_I = \tilde{B}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{z}} \times \hat{\mathbf{x}}$$

$$(20) \quad = \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}}$$

We'll have a reflected wave ($\hat{\mathbf{k}} = -\hat{\mathbf{z}}$) and a transmitted wave ($\hat{\mathbf{k}} = \hat{\mathbf{z}}$), so

$$(21) \quad \tilde{\mathbf{E}}_R = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}})$$

$$(22) \quad \tilde{\mathbf{B}}_R = -\tilde{B}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{z}} \times (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}})$$

$$(23) \quad = \frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} (\sin \theta_R \hat{\mathbf{x}} - \cos \theta_R \hat{\mathbf{y}})$$

$$(24) \quad \tilde{\mathbf{E}}_T = \tilde{E}_{0T} e^{i(k_2 z - \omega t)} (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}})$$

$$(25) \quad \tilde{\mathbf{B}}_T = \tilde{B}_{0T} e^{i(k_2 z - \omega t)} \hat{\mathbf{z}} \times (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}})$$

$$(26) \quad = \frac{1}{v_2} \tilde{E}_{0T} e^{i(k_2 z - \omega t)} (-\sin \theta_T \hat{\mathbf{x}} + \cos \theta_T \hat{\mathbf{y}})$$

where θ_R and θ_T are the angles of polarization for the reflected and transmitted waves.

Since there are no components of the fields perpendicular to the boundary at $z = 0$, 14 and 15 tell us nothing. Applying 16 to the x and y components, we have

$$(27) \quad \tilde{E}_{0I} \hat{\mathbf{x}} + \tilde{E}_{0R} (\cos \theta_R \hat{\mathbf{x}} + \sin \theta_R \hat{\mathbf{y}}) = \tilde{E}_{0T} (\cos \theta_T \hat{\mathbf{x}} + \sin \theta_T \hat{\mathbf{y}})$$

$$(28) \quad \tilde{E}_{0I} + \tilde{E}_{0R} \cos \theta_R = \tilde{E}_{0T} \cos \theta_T$$

$$(29) \quad \tilde{E}_{0R} \sin \theta_R = \tilde{E}_{0T} \sin \theta_T$$

From 17 we have

$$(30) \quad \frac{1}{\mu_1 v_1} \tilde{E}_{0R} \sin \theta_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin \theta_T$$

$$(31) \quad \frac{1}{\mu_1 v_1} (\tilde{E}_{0I} - \tilde{E}_{0R} \cos \theta_R) = \frac{1}{\mu_2 v_2} \tilde{E}_{0T} \cos \theta_T$$

Substituting from 29 into 30 we have

$$(32) \quad \frac{1}{\mu_1 v_1} \tilde{E}_{0T} \sin \theta_T = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin \theta_T$$

The only way this equation can be satisfied is if $\sin \theta_T = 0$ so from 29 we conclude that $\sin \theta_R = 0$ as well. Thus θ_T and θ_R are either 0 or π , so the cosines are ± 1 . Thus the plane of polarization is the same for the reflected and transmitted waves as for the incident wave.

If we define

$$(33) \quad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2}$$

we can write 31 as

$$(34) \quad \tilde{E}_{0I} - \tilde{E}_{0R} \cos \theta_R = \beta \tilde{E}_{0T} \cos \theta_T$$

Adding this to 28 we get

$$(35) \quad \tilde{E}_{0I} = \frac{1+\beta}{2} \tilde{E}_{0T} \cos \theta_T$$

$$(36) \quad \tilde{E}_{0T} = \pm \frac{2}{1+\beta} \tilde{E}_{0I}$$

Subtracting 34 from 28 we get

$$(37) \quad \tilde{E}_{0R} = \pm \frac{1-\beta}{2} \tilde{E}_{0T} = \pm \frac{1-\beta}{1+\beta} \tilde{E}_{0I}$$

Thus the reflected and transmitted waves are either in phase or half a cycle out of phase with the incident wave.

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