DECAY TIME FOR FREE CHARGE IN A CONDUCTOR

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.18a.

When we looked at conductors in electrostatics, we saw that any free charge in a conductor must reside on the surface. In electrodynamics, however, we can place some free charge inside a conductor and then watch it (figuratively) move to the surface. How long does this migration take?

We start with Maxwell's equations within matter:

$$\begin{aligned}
\nabla\cdot\mathbf{D} &= \rho_f && (1)\\
\nabla\cdot\mathbf{B} &= 0 && (2)\\
\nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t} && (3)\\
\nabla\times\mathbf{H} &= \mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t} && (4)
\end{aligned}$$

For linear media, $\mathbf{D}=\varepsilon\mathbf{E}$ and $\mathbf{H}=\mathbf{B}/\mu$, so these equations become

$$\begin{aligned}
\nabla\cdot\mathbf{E} &= \frac{1}{\varepsilon}\rho_f && (5)\\
\nabla\cdot\mathbf{B} &= 0 && (6)\\
\nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t} && (7)\\
\nabla\times\mathbf{B} &= \mu\mathbf{J}_f+\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} && (8)
\end{aligned}$$

Within a conductor with conductivity σ (*not* surface charge density!) Ohm's law can be written as

$$\mathbf{J}_f=\sigma\mathbf{E} \tag{9}$$

so Maxwell's equations become

$$\nabla\cdot\mathbf{E} = \frac{1}{\varepsilon}\rho_{f} \tag{10}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{11}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{12}$$

$$\nabla\times\mathbf{B} = \mu\sigma\mathbf{E}+\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \tag{13}$$

The conservation of charge (continuity condition) says

$$\nabla\cdot\mathbf{J}_{f} = -\frac{\partial\rho_{f}}{\partial t} \tag{14}$$

so from 9 and 10 we get

$$\frac{\partial\rho_{f}}{\partial t} = -\sigma\nabla\cdot\mathbf{E} = -\frac{\sigma}{\varepsilon}\rho_{f} \tag{15}$$

For a fixed location within the conductor we can integrate this with respect to time to get

$$\rho_{f}(t) = \rho_{f}(0)\,e^{-\sigma t/\varepsilon} \tag{16}$$

That is, the free charge density decays exponentially with a characteristic time $\tau=\varepsilon/\sigma$.

Example. For glass, the conductivity is around 10^{-11} S m^{-1} (S stands for Siemens, which is the SI unit of conductance, where 1 S $=$ 1 kg^{-1}m^{-2}s^{3}A^{2}) and the permittivity is $4.7\varepsilon_{0}=4.16\times10^{-11}$m^{-3}kg^{-1}s^{4}A^{2} so the charge would move to the surface in a time of about

$$\tau = 4.16 \text{ s} \tag{17}$$

Some forms of glass have much smaller conductivities, so the migration time would be correspondingly larger.

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