DECAY TIME FOR FREE CHARGE IN A CONDUCTOR

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.18a.

When we looked at conductors in electrostatics, we saw that any free charge in a conductor must reside on the surface. In electrodynamics, however, we can place some free charge inside a conductor and then watch it (figuratively) move to the surface. How long does this migration take?

We start with Maxwell's equations within matter:

$$\nabla\cdot\mathbf{D} = \rho_f \tag{1}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{2}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{3}$$

$$\nabla\times\mathbf{H} = \mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t} \tag{4}$$

For linear media, $\mathbf{D}=\varepsilon\mathbf{E}$ and $\mathbf{H}=\mathbf{B}/\mu$, so these equations become

$$\nabla\cdot\mathbf{E} = \frac{1}{\varepsilon}\rho_f \tag{5}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{6}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{7}$$

$$\nabla\times\mathbf{B} = \mu\mathbf{J}_f+\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \tag{8}$$

Within a conductor with conductivity σ (*not* surface charge density!) Ohm's law can be written as

$$\mathbf{J}_f=\sigma\mathbf{E} \tag{9}$$

so Maxwell's equations become

$$\nabla\cdot\mathbf{E} = \frac{1}{\varepsilon}\rho_f \tag{10}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{11}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{12}$$

$$\nabla\times\mathbf{B} = \mu\sigma\mathbf{E}+\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \tag{13}$$

The conservation of charge (continuity condition) says

$$\nabla\cdot\mathbf{J}_f = -\frac{\partial\rho_f}{\partial t} \tag{14}$$

so from 9 and 10 we get

$$\frac{\partial\rho_f}{\partial t} = -\sigma\nabla\cdot\mathbf{E} = -\frac{\sigma}{\varepsilon}\rho_f \tag{15}$$

For a fixed location within the conductor we can integrate this with respect to time to get

$$\rho_f(t) = \rho_f(0)\,e^{-\sigma t/\varepsilon} \tag{16}$$

That is, the free charge density decays exponentially with a characteristic time $\tau = \varepsilon/\sigma$.

Example. For glass, the conductivity is around 10^{-11} S m^{-1} (S stands for Siemens, which is the SI unit of conductance, where $1\text{ S} = 1\text{ kg}^{-1}\text{m}^{-2}\text{s}^{3}\text{A}^{2}$) and the permittivity is $4.7\varepsilon_0 = 4.16\times10^{-11}\text{m}^{-3}\text{kg}^{-1}\text{s}^{4}\text{A}^{2}$ so the charge would move to the surface in a time of about

$$\tau = 4.16\text{ s} \tag{17}$$

Some forms of glass have much smaller conductivities, so the migration time would be correspondingly larger.

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