

DECAY TIME FOR FREE CHARGE IN A CONDUCTOR

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.18a.

When we looked at conductors in electrostatics, we saw that any free charge in a conductor must reside on the surface. In electrodynamics, however, we can place some free charge inside a conductor and then watch it (figuratively) move to the surface. How long does this migration take?

We start with Maxwell's equations within matter:

$$\nabla \cdot \mathbf{D} = \rho_f \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

For linear media, $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$, so these equations become

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon} \rho_f \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (7)$$

$$\nabla \times \mathbf{B} = \mu \mathbf{J}_f + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (8)$$

Within a conductor with conductivity σ (*not* surface charge density!) Ohm's law can be written as

$$\mathbf{J}_f = \sigma \mathbf{E} \quad (9)$$

so Maxwell's equations become

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon} \rho_f \quad (10)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (11)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (12)$$

$$\nabla \times \mathbf{B} = \mu\sigma \mathbf{E} + \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (13)$$

The conservation of charge (continuity condition) says

$$\nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t} \quad (14)$$

so from 9 and 10 we get

$$\frac{\partial \rho_f}{\partial t} = -\sigma \nabla \cdot \mathbf{E} = -\frac{\sigma}{\epsilon} \rho_f \quad (15)$$

For a fixed location within the conductor we can integrate this with respect to time to get

$$\rho_f(t) = \rho_f(0) e^{-\sigma t/\epsilon} \quad (16)$$

That is, the free charge density decays exponentially with a characteristic time $\tau = \epsilon/\sigma$.

Example. For glass, the conductivity is around $10^{-11} \text{ S m}^{-1}$ (S stands for Siemens, which is the SI unit of conductance, where $1 \text{ S} = 1 \text{ kg}^{-1} \text{ m}^{-2} \text{ s}^3 \text{ A}^2$) and the permittivity is $4.7\epsilon_0 = 4.16 \times 10^{-11} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$ so the charge would move to the surface in a time of about

$$\tau = 4.16 \text{ s} \quad (17)$$

Some forms of glass have much smaller conductivities, so the migration time would be correspondingly larger.

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