

## DECAY TIME FOR FREE CHARGE IN A CONDUCTOR

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.18a.

When we looked at conductors in electrostatics, we saw that any free charge in a conductor must reside on the surface. In electrodynamics, however, we can place some free charge inside a conductor and then watch it (figuratively) move to the surface. How long does this migration take?

We start with Maxwell's equations within matter:

$$(0.1) \quad \nabla \cdot \mathbf{D} = \rho_f$$

$$(0.2) \quad \nabla \cdot \mathbf{B} = 0$$

$$(0.3) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(0.4) \quad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

For linear media,  $\mathbf{D} = \epsilon \mathbf{E}$  and  $\mathbf{H} = \mathbf{B}/\mu$ , so these equations become

$$(0.5) \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon} \rho_f$$

$$(0.6) \quad \nabla \cdot \mathbf{B} = 0$$

$$(0.7) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(0.8) \quad \nabla \times \mathbf{B} = \mu \mathbf{J}_f + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

Within a conductor with conductivity  $\sigma$  (*not* surface charge density!) Ohm's law can be written as

$$(0.9) \quad \mathbf{J}_f = \sigma \mathbf{E}$$

so Maxwell's equations become

$$(0.10) \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon} \rho_f$$

$$(0.11) \quad \nabla \cdot \mathbf{B} = 0$$

$$(0.12) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(0.13) \quad \nabla \times \mathbf{B} = \mu \sigma \mathbf{E} + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

The conservation of charge (continuity condition) says

$$(0.14) \quad \nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}$$

so from 0.9 and 0.10 we get

$$(0.15) \quad \frac{\partial \rho_f}{\partial t} = -\sigma \nabla \cdot \mathbf{E} = -\frac{\sigma}{\epsilon} \rho_f$$

For a fixed location within the conductor we can integrate this with respect to time to get

$$(0.16) \quad \rho_f(t) = \rho_f(0) e^{-\sigma t/\epsilon}$$

That is, the free charge density decays exponentially with a characteristic time  $\tau = \epsilon/\sigma$ .

**Example.** For glass, the conductivity is around  $10^{-11} \text{ S m}^{-1}$  (S stands for Siemens, which is the SI unit of conductance, where  $1 \text{ S} = 1 \text{ kg}^{-1} \text{ m}^{-2} \text{ s}^3 \text{ A}^2$ ) and the permittivity is  $4.7\epsilon_0 = 4.16 \times 10^{-11} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$  so the charge would move to the surface in a time of about

$$(0.17) \quad \tau = 4.16 \text{ s}$$

Some forms of glass have much smaller conductivities, so the migration time would be correspondingly larger.

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