SKIN DEPTH OF WATER AND METALS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.19a-b.

Electromagnetic waves in a conductor (where there is free current but no free charge) can be written as

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i\left(\tilde{k}z-\omega t\right)} \tag{0.1}$$

$$\tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i\left(\tilde{k}z-\omega t\right)} \tag{0.2}$$

where the wave vector is complex:

(0.3)

$$\tilde{k} = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1}+i\frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1} \equiv k+i\kappa$$

For a poor conductor, the conductivity σ is small, so for large enough frequencies $\sigma \ll \varepsilon\omega$ and we can approximate κ by

$$\kappa \approx \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{1+\frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2-1} \tag{0.4}$$

$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \tag{0.5}$$

Since the imaginary part of $\tilde{k}$ governs the attenuation of the wave as it penetrates the material, the skin depth for a poor conductor is

$$d = \frac{1}{\kappa} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}} \tag{0.6}$$

For pure (deionized) water $\sigma = 5.5\times10^{-6}\text{S m}^{-1}$ and $\varepsilon = 80.1\varepsilon_0$ (at 20°C) (we can take $\mu \approx \mu_0$) so the skin depth of water is

$$d = 8635\text{ m} \tag{0.7}$$

Because the skin depth is so large, water is transparent.

For a good conductor, $\sigma \gg \varepsilon\omega$ and we can approximate

$$\kappa \approx \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\mu\sigma\omega}{2}} \approx k \tag{0.8}$$

so the skin depth is

$$d = \sqrt{\frac{2}{\mu\sigma\omega}} \approx \frac{1}{k} = \frac{2\pi}{\lambda} \tag{0.9}$$

where λ is the wavelength within the material. For a typical metal, $\sigma \approx 10^7 \text{S m}^{-1}$ and $\mu \approx \mu_0$ so the skin depth at visible frequencies $\omega \approx 10^{15}\text{s}^{-1}$ is

$$d \approx 1.26 \times 10^{-8}\text{m} \tag{0.10}$$

With a skin depth this small, even a thin film of metal is effectively impervious to any penetration by visible light.

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