

RECTANGULAR WAVE GUIDES: TE MODES AND THE CUTOFF FREQUENCY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.28.

The possible TE modes in a rectangular wave guide are given by

$$(1) \quad B_z(x,y) = B_0 \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$$

where m and n are integers and a and b are the dimensions of the rectangle, with $a \geq b$. These values are related to the wave number by

$$(2) \quad k = \sqrt{\frac{\omega^2}{c^2} - \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}$$

so any modes where

$$(3) \quad \omega^2 < c^2 \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$$

are not allowed, as they would give an imaginary value of k . The frequency

$$(4) \quad \omega_{mn} = \pi c \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

is therefore the cutoff frequency for the mode TE_{mn} .

As an example, suppose we had a wave guide with $a = 2.28$ cm, $b = 1.01$ cm and a frequency of $\omega = 1.70 \times 10^{10}$ Hz. Then the condition for a real k is

$$(5) \quad \frac{\omega^2}{\pi^2 c^2} \geq \frac{m^2}{a^2} + \frac{n^2}{b^2}$$

$$(6) \quad \frac{4v^2}{c^2} \geq \frac{m^2}{a^2} + \frac{n^2}{b^2}$$

$$(7) \quad 1.284 \times 10^4 \geq 10^4 \left(\frac{m^2}{5.2} + \frac{n^2}{1.02} \right)$$

$$(8) \quad 1.02m^2 + 5.2n^2 \leq 6.813$$

Since at least one of m and n must be non-zero, the allowed modes are TE₁₀, TE₂₀, TE₀₁ and TE₁₁ (where the first subscript is m and the second is n).

If we wish to allow only one mode, this would have to be TE₁₀ (since that gives the lowest value on the LHS). The angular frequency range that would do this is between ω_{10} and ω_{20} , that is

$$(9) \quad \omega_{10} = \frac{\pi c}{a} \leq \omega < \frac{2\pi c}{a} = \omega_{20}$$

For the values above

$$(10) \quad \omega_{10} = 4.134 \times 10^{10} \text{ s}^{-1}$$

$$(11) \quad \omega_{20} = 8.268 \times 10^{10} \text{ s}^{-1}$$

The corresponding frequencies and wavelengths are

$$(12) \quad \nu_{10} = \frac{\omega_{10}}{2\pi} = 6.58 \times 10^9 \text{ Hz}$$

$$(13) \quad \nu_{20} = 1.32 \times 10^{10} \text{ Hz}$$

$$(14) \quad \lambda_{10} = \frac{c}{\nu_{10}} = 0.0456 \text{ m}$$

$$(15) \quad \lambda_{20} = 0.0228 \text{ m}$$