

## RECTANGULAR WAVE GUIDE: TRANSVERSE MAGNETIC MODES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 9.30.

We can work out the theory of a transverse magnetic (TM) wave in a rectangular wave guide of dimensions  $a$  in the  $x$  direction and  $b$  in the  $y$  direction, in the same way as for the TE wave. In a TM wave the component  $B_z$  parallel to the axis of the wave guide is zero, so we have only a single wave equation to solve.

$$(0.1) \quad \left[ \partial_{xx} + \partial_{yy} + \frac{\omega^2}{c^2} - k^2 \right] E_z = 0$$

As this is identical to the equation for TE waves, we have the same solution:

$$(0.2) \quad E_z = X(x)Y(y)$$

$$(0.3) \quad X(x) = A \sin k_x x + B \cos k_x x$$

$$(0.4) \quad Y(y) = C \sin k_y y + D \cos k_y y$$

The boundary conditions are different here, however, as we require

$$(0.5) \quad \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = 0$$

If the wave guide is a perfect conductor, then  $\mathbf{E} = 0$  inside it, so  $\mathbf{E}^{\parallel} = 0$  at all boundaries of the guide. In particular,  $E_z = 0$  for  $x = 0, a$  and  $y = 0, b$ . This means  $B = D = 0$ , so

$$(0.6) \quad E_z(x, y) = E_0 \sin k_x x \sin k_y y$$

and

$$(0.7) \quad k_x = \frac{m\pi}{a}$$

$$(0.8) \quad k_y = \frac{n\pi}{b}$$

Because  $E_z$  uses sines rather than cosines, the integers  $m$  and  $n$  must both be non-zero in order for the wave to exist at all, so the lowest TM mode is  $\text{TM}_{11}$ .

The wave number has the same form as in the TE case:

$$(0.9) \quad k = \sqrt{\frac{\omega^2}{c^2} - \pi^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}$$

$$(0.10) \quad \equiv \frac{1}{c} \sqrt{\omega^2 - \omega_{mn}^2}$$

$$(0.11) \quad \omega_{mn} = c\pi \sqrt{\left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}$$

The phase and group velocities are the same as for TE waves

$$(0.12) \quad v_p = \frac{\omega}{k}$$

$$(0.13) \quad v_g = \frac{d\omega}{dk} = c \sqrt{1 - \frac{\omega_{mn}^2}{\omega^2}}$$

The ratio of lowest cutoff frequencies is

$$(0.14) \quad \frac{\omega_{11}^{TM}}{\omega_{10}^{TE}} = \frac{\pi c \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}{\pi c/a} = \frac{1}{b} \sqrt{a^2 + b^2}$$