

POTENTIALS FOR A POINT CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.3.

As a simple (though unusual) example of specifying a system through electric and magnetic potentials suppose we have

$$(1) \quad V(\mathbf{r}, t) = 0$$
$$(2) \quad \mathbf{A}(\mathbf{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{\mathbf{r}}$$

These potentials give rise to the fields

$$(3) \quad \mathbf{B} = \nabla \times \mathbf{A} = 0$$
$$(4) \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

The expression for \mathbf{E} is just that of a point charge q at the origin, while a zero magnetic field indicates that there is no current. If we want to be pedantic, we can also get these results from the potentials. For the charge density, we had

$$(5) \quad \nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) = -\frac{\rho}{\epsilon_0}$$

Using the formula

$$(6) \quad \nabla \cdot \left(\frac{1}{r^2} \hat{\mathbf{r}} \right) = 4\pi \delta_3(\mathbf{r})$$

we get

$$(7) \quad \nabla \cdot \mathbf{A} = -\frac{qt}{\epsilon_0} \delta_3(\mathbf{r})$$

$$(8) \quad \rho = q \delta_3(\mathbf{r})$$

For the current, we can use the equation

$$(9) \quad \nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J} - \mu_0 \epsilon_0 \nabla \frac{\partial V}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}$$

and we see that all the terms not involving \mathbf{J} are zero, so $\mathbf{J} = 0$ as well. Thus this is a bizarre way of writing potentials for a point charge. This illustrates that the potentials giving rise to a particular charge and current distribution are not unique.

PINGBACKS

Pingback: [Gauge transformations in electrodynamics](#)

Pingback: [Coulomb and Lorenz gauges](#)