POTENTIALS FOR A POINT CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.3.

As a simple (though unusual) example of specifying a system through electric and magnetic potentials suppose we have

$$V(\mathbf{r},t) = 0 \tag{0.1}$$

$$\mathbf{A}(\mathbf{r},t) = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{\mathbf{r}} \tag{0.2}$$

These potentials give rise to the fields

$$\mathbf{B} = \nabla\times\mathbf{A} = 0 \tag{0.3}$$

$$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \tag{0.4}$$

The expression for $\mathbf{E}$ is just that of a point charge q at the origin, while a zero magnetic field indicates that there is no current. If we want to be pedantic, we can also get these results from the potentials. For the charge density, we had

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\frac{\rho}{\epsilon_0} \tag{0.5}$$

Using the formula

$$\nabla\cdot\left(\frac{1}{r^2}\hat{\mathbf{r}}\right) = 4\pi\delta_3(\mathbf{r}) \tag{0.6}$$

we get

$$\nabla\cdot\mathbf{A} = -\frac{qt}{\epsilon_0}\delta_3(\mathbf{r}) \tag{0.7}$$

$$\rho = q\delta_3(\mathbf{r}) \tag{0.8}$$

For the current, we can use the equation

$$\nabla\times(\nabla\times\mathbf{A})=\mu_0\mathbf{J}-\mu_0\varepsilon_0\nabla\frac{\partial V}{\partial t}-\mu_0\varepsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2} \tag{0.9}$$

and we see that all the terms not involving $\mathbf{J}$ are zero, so $\mathbf{J}=0$ as well. Thus this is a bizarre way of writing potentials for a point charge. This illustrates that the potentials giving rise to a particular charge and current distribution are not unique.

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