

POTENTIALS FOR AN ELECTROMAGNETIC WAVE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.4.

We can define an electromagnetic wave in terms of electric and magnetic potentials as follows. Using rectangular coordinates, let

$$V = 0 \quad (1)$$

$$\mathbf{A} = A_0 \sin(kx - \omega t) \hat{\mathbf{y}} \quad (2)$$

These potentials give rise to the fields

$$\mathbf{B} = \nabla \times \mathbf{A} = A_0 k \cos(kx - \omega t) \hat{\mathbf{z}} \quad (3)$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = A_0 \omega \cos(kx - \omega t) \hat{\mathbf{y}} \quad (4)$$

We can check that these fields satisfy Maxwell's equations in vacuum. First,

$$\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0 \quad (5)$$

For the curls, we have

$$\nabla \times \mathbf{E} = -A_0 k \omega \sin(kx - \omega t) \hat{\mathbf{z}} = -\frac{\partial \mathbf{B}}{\partial t} \quad (6)$$

$$\nabla \times \mathbf{B} = A_0 k^2 \sin(kx - \omega t) \hat{\mathbf{y}} \quad (7)$$

The second equation should be equal to $\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$ so

$$A_0 k^2 \sin(kx - \omega t) \hat{\mathbf{y}} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (8)$$

$$= \frac{1}{c^2} A_0 \omega^2 \sin(kx - \omega t) \hat{\mathbf{y}} \quad (9)$$

which can be true only if

$$k^2 = \frac{\omega^2}{c^2} \quad (10)$$

which is the usual relation between wave number k and angular frequency ω .

PINGBACKS

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