

## POTENTIALS FOR AN ELECTROMAGNETIC WAVE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.4.

We can define an electromagnetic wave in terms of electric and magnetic potentials as follows. Using rectangular coordinates, let

$$(0.1) \quad V = 0$$

$$(0.2) \quad \mathbf{A} = A_0 \sin(kx - \omega t) \hat{\mathbf{y}}$$

These potentials give rise to the fields

$$(0.3) \quad \mathbf{B} = \nabla \times \mathbf{A} = A_0 k \cos(kx - \omega t) \hat{\mathbf{z}}$$

$$(0.4) \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = A_0 \omega \cos(kx - \omega t) \hat{\mathbf{y}}$$

We can check that these fields satisfy Maxwell's equations in vacuum. First,

$$(0.5) \quad \nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0$$

For the curls, we have

$$(0.6) \quad \nabla \times \mathbf{E} = -A_0 k \omega \sin(kx - \omega t) \hat{\mathbf{z}} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(0.7) \quad \nabla \times \mathbf{B} = A_0 k^2 \sin(kx - \omega t) \hat{\mathbf{y}}$$

The second equation should be equal to  $\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$  so

$$(0.8) \quad A_0 k^2 \sin(kx - \omega t) \hat{\mathbf{y}} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

$$(0.9) \quad = \frac{1}{c^2} A_0 \omega^2 \sin(kx - \omega t) \hat{\mathbf{y}}$$

which can be true only if

$$(0.10) \quad k^2 = \frac{\omega^2}{c^2}$$

which is the usual relation between wave number  $k$  and angular frequency  $\omega$ .

PINGBACKS

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