POTENTIALS FOR AN ELECTROMAGNETIC WAVE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.4.

We can define an electromagnetic wave in terms of electric and magnetic potentials as follows. Using rectangular coordinates, let

$$V = 0 \tag{1}$$

$$\mathbf{A} = A_0 \sin(kx - \omega t)\,\hat{\mathbf{y}} \tag{2}$$

These potentials give rise to the fields

$$\mathbf{B} = \nabla\times\mathbf{A} = A_0 k \cos(kx - \omega t)\,\hat{\mathbf{z}} \tag{3}$$

$$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t} = A_0 \omega \cos(kx - \omega t)\,\hat{\mathbf{y}} \tag{4}$$

We can check that these fields satisfy Maxwell's equations in vacuum. First,

$$\nabla\cdot\mathbf{E} = \nabla\cdot\mathbf{B} = 0 \tag{5}$$

For the curls, we have

$$\nabla\times\mathbf{E} = -A_0 k\omega \sin(kx - \omega t)\,\hat{\mathbf{z}} = -\frac{\partial\mathbf{B}}{\partial t} \tag{6}$$

$$\nabla\times\mathbf{B} = A_0 k^2 \sin(kx - \omega t)\,\hat{\mathbf{y}} \tag{7}$$

The second equation should be equal to $\mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$ so

$$A_0 k^2 \sin(kx - \omega t)\,\hat{\mathbf{y}} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} \tag{8}$$

$$= \frac{1}{c^2} A_0 \omega^2 \sin(kx - \omega t)\,\hat{\mathbf{y}} \tag{9}$$

which can be true only if

$$k^2 = \frac{\omega^2}{c^2} \tag{10}$$

which is the usual relation between wave number k and angular frequency ω.

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