

LORENZ GAUGE IS ALWAYS POSSIBLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.7.

The Lorenz gauge is defined by setting

$$\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \quad (1)$$

but is it always possible to do this? We can show that it is using a similar technique to that for the Coulomb gauge. We want a function λ which we can use to transform some arbitrary potentials \mathbf{A}' and V so that that \mathbf{A} satisfies 1 as follows:

$$\mathbf{A} = \mathbf{A}' + \nabla \lambda \quad (2)$$

$$V = V' - \frac{\partial \lambda}{\partial t} \quad (3)$$

Taking the divergence of both sides, we get

$$\nabla \cdot \mathbf{A} = \nabla \cdot \mathbf{A}' + \nabla^2 \lambda \quad (4)$$

$$= -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \quad (5)$$

$$= -\mu_0 \epsilon_0 \frac{\partial V'}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 \lambda}{\partial t^2} \quad (6)$$

Combining the first and last equations we get

$$\nabla^2 \lambda - \mu_0 \epsilon_0 \frac{\partial^2 \lambda}{\partial t^2} = -\mu_0 \epsilon_0 \frac{\partial V'}{\partial t} - \nabla \cdot \mathbf{A}' \quad (7)$$

$$\square^2 \lambda = -\mu_0 \epsilon_0 \frac{\partial V'}{\partial t} - \nabla \cdot \mathbf{A}' \quad (8)$$

That is, λ is the solution of the wave equation with a driving term, which we can, in principle, always solve (although it may not be easy!). Therefore we can always find the function λ to convert an arbitrary pair of potentials \mathbf{A}' and V' to the Lorenz gauge.

In general (not necessarily in the Lorenz gauge), we could always set $V = 0$ by choosing

$$\frac{\partial \lambda}{\partial t} = V' \quad (9)$$

$$\lambda = \int_0^t V'(\mathbf{r}, t') dt' \quad (10)$$

We can't always choose $\mathbf{A} = 0$ however, since $\mathbf{B} = \nabla \times \mathbf{A}$ and if the magnetic field is non-zero, then \mathbf{A} can't be zero everywhere.