

RETARDED POTENTIAL OF A WIRE LOOP

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.10.

One more example of calculating the retarded potential. We have a loop of wire in the following shape. It extends along the x axis from $-b$ to $-a$, then in a semicircular loop of radius a clockwise around to $x = +a$, then along the x axis from $+a$ to $+b$, then in a semicircular loop of radius b back to $x = -b$. A linearly increasing current

$$I(t) = kt \tag{1}$$

flows through the loop in the direction given above. Assuming the wire is electrically neutral, $V = 0$ so our job is to find \mathbf{A} .

Calculating \mathbf{A} in general is a complex task, so we'll look only at the value of \mathbf{A} at the origin. Consider first the inner loop of radius a . All points on this loop are at the same distance a from the origin, so the retarded time is the same for all points on the loop. Since the current goes clockwise around the semicircle, the contribution to \mathbf{A} is

$$\mathbf{A}_a = \frac{\mu_0}{4\pi} \int_{\pi}^0 k \left(t - \frac{a}{c} \right) \frac{\hat{\theta} a}{a} d\theta \tag{2}$$

$$= -\frac{\mu_0}{4\pi} k \left(t - \frac{a}{c} \right) \int_0^{\pi} (-\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}}) d\theta \tag{3}$$

$$= \frac{\mu_0}{2\pi} k \left(t - \frac{a}{c} \right) \hat{\mathbf{x}} \tag{4}$$

We get a similar expression for the loop around the outer semicircle except this time the current flows counterclockwise so the sign is reversed:

$$\mathbf{A}_b = -\frac{\mu_0}{2\pi} k \left(t - \frac{b}{c} \right) \hat{\mathbf{x}} \tag{5}$$

Adding these two together we get

$$\mathbf{A}_{ab} = \frac{\mu_0}{2\pi} k \frac{b-a}{c} \hat{\mathbf{x}} \tag{6}$$

The contributions from each of the two horizontal segments are equal, so for these two segments we have

$$\mathbf{A}_x = 2 \frac{\mu_0}{4\pi} k \hat{\mathbf{x}} \int_a^b \frac{t - \frac{x}{c}}{x} dx \quad (7)$$

$$= \frac{\mu_0}{2\pi} k \hat{\mathbf{x}} \left[t \ln \frac{b}{a} - \frac{b-a}{c} \right] \quad (8)$$

The total potential is then

$$\mathbf{A}(0, t) = \mathbf{A}_{ab} + \mathbf{A}_x \quad (9)$$

$$= \frac{\mu_0}{2\pi} k \hat{\mathbf{x}} t \ln \frac{b}{a} \quad (10)$$

Because we have the potential at only a single point in space, we can't calculate any of its derivatives, so we can't calculate $\mathbf{B} = \nabla \times \mathbf{A}$. However we can calculate \mathbf{E} :

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad (11)$$

$$= -\frac{\mu_0}{2\pi} k \hat{\mathbf{x}} \ln \frac{b}{a} \quad (12)$$

The electric field is constant in time at the origin. An electrically neutral wire can produce an electric field since the changing current induces a changing magnetic field which in turn produces an electric field.