

## LIÉNARD-WIECHERT POTENTIALS FOR A CHARGE MOVING WITH CONSTANT VELOCITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problems 10.14.

Griffiths shows in his example 10.3 that the Liénard-Wiechert potentials for a point charge  $q$  moving at constant velocity  $\mathbf{v}$  that passes through the origin at time  $t = 0$  are

$$(0.1) \quad V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{\sqrt{(c^2t - \mathbf{r} \cdot \mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2t^2)}}$$

$$(0.2) \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{qc\mathbf{v}}{\sqrt{(c^2t - \mathbf{r} \cdot \mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2t^2)}}$$

These potentials can be expressed in a simpler form by defining the vector

$$(0.3) \quad \mathbf{R} \equiv \mathbf{r} - \mathbf{v}t$$

We can eliminate  $\mathbf{r}$  from 0.1 as follows.

$$(0.4) \quad \mathbf{R} \cdot \mathbf{v} = \mathbf{r} \cdot \mathbf{v} - v^2t$$

$$(0.5) \quad \mathbf{r} \cdot \mathbf{v} = \mathbf{R} \cdot \mathbf{v} + v^2t$$

$$(0.6) \quad R^2 = r^2 + v^2t^2 - 2\mathbf{r} \cdot \mathbf{v}t$$

$$(0.7) \quad r^2 = R^2 - v^2t^2 + 2\mathbf{r} \cdot \mathbf{v}t$$

$$(0.8) \quad = R^2 + v^2t^2 + 2\mathbf{R} \cdot \mathbf{v}t$$

$$(0.9) \quad (c^2t - \mathbf{r} \cdot \mathbf{v})^2 = (c^2t - \mathbf{R} \cdot \mathbf{v} - v^2t)^2$$

$$(0.10) \quad (c^2 - v^2)(r^2 - c^2t^2) = (c^2 - v^2)(R^2 + v^2t^2 + 2\mathbf{R} \cdot \mathbf{v}t - c^2t^2)$$

Adding the last two RHSs together and cancelling terms, we get

$$(0.11) \quad (c^2t - \mathbf{r} \cdot \mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2t^2) = (\mathbf{R} \cdot \mathbf{v})^2 + (c^2 - v^2)R^2$$

If  $\theta$  is the angle between  $\mathbf{R}$  and  $\mathbf{v}$ , then this becomes

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$$(0.12) \quad (\mathbf{R} \cdot \mathbf{v})^2 + (c^2 - v^2) R^2 = R^2 (c^2 - v^2 (1 - \cos^2 \theta))$$

$$(0.13) \quad = R^2 c^2 \left( 1 - \frac{v^2}{c^2} \sin^2 \theta \right)$$

Inserting this back into 0.1 we get

$$(0.14) \quad V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R \sqrt{\left( 1 - \frac{v^2}{c^2} \sin^2 \theta \right)}}$$

Note that  $R$  and  $\theta$  are both functions of time since they vary as the charge moves. For non-relativistic speeds,  $v \ll c$  and the formula reduces to the Coulomb potential from electrostatics:

$$(0.15) \quad V = \frac{q}{4\pi\epsilon_0 R}$$