

FIELDS OF A POINT CHARGE MOVING IN ONE DIMENSION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.18.

Here's a simple example of calculating the fields due to a moving point charge. Suppose we have a charge constrained to move on the x axis in the $+x$ direction, so that $\mathbf{v} = v\hat{\mathbf{x}}$.

The fields are

$$(0.1) \quad \mathbf{E}(\mathbf{r}, t) = \frac{q\mathbf{r}}{4\pi\epsilon_0(\mathbf{r}\cdot\mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \mathbf{r} \times (\mathbf{u} \times \mathbf{a})]$$

$$(0.2) \quad \mathbf{B}(\mathbf{r}, t) = \frac{1}{c}\hat{\mathbf{t}} \times \mathbf{E}(\mathbf{r}, t)$$

where

$$(0.3) \quad \mathbf{r} \equiv \mathbf{r} - \mathbf{w}(t_r)$$

$$(0.4) \quad \mathbf{u} \equiv c\hat{\mathbf{t}} - \mathbf{v}$$

and $\mathbf{w}(t_r)$ is the particle's position at the retarded time. If the observer is to the right of the particle then

$$(0.5) \quad \mathbf{r} = +r\hat{\mathbf{x}}$$

$$(0.6) \quad \mathbf{u} = (c - v)\hat{\mathbf{x}}$$

Since the motion is constrained to the x axis, any velocity and acceleration must be parallel, so $\mathbf{u} \times \mathbf{a} = 0$ and we have

$$(0.7) \quad \mathbf{E} = \frac{q\mathbf{r}}{4\pi\epsilon_0(\mathbf{r}\cdot\mathbf{u})^3} (c^2 - v^2)\mathbf{u}$$

$$(0.8) \quad = \frac{q(c^2 - v^2)}{4\pi\epsilon_0(c - v)^3 r^2} (c - v)\hat{\mathbf{x}}$$

$$(0.9) \quad = \frac{q(c + v)}{4\pi\epsilon_0(c - v)r^2}\hat{\mathbf{x}}$$

Because $\hat{\mathbf{t}}$ is parallel to \mathbf{E} , $\mathbf{B} = 0$ from 0.2.

If the observer is to the left of the charge, then

$$(0.10) \quad \mathbf{r} = -r\hat{\mathbf{x}}$$

$$(0.11) \quad \mathbf{u} = -(c+v)\hat{\mathbf{x}}$$

$$(0.12) \quad \mathbf{E} = \frac{q\mathbf{r}}{4\pi\epsilon_0(\mathbf{r}\cdot\mathbf{u})^3}(c^2-v^2)\mathbf{u}$$

$$(0.13) \quad = -\frac{q(c^2-v^2)}{4\pi\epsilon_0(c+v)^3r^2}(c+v)\hat{\mathbf{x}}$$

$$(0.14) \quad = -\frac{q(c-v)}{4\pi\epsilon_0(c+v)r^2}\hat{\mathbf{x}}$$

$$(0.15) \quad \mathbf{B} = 0$$