FIELDS OF A POINT CHARGE MOVING IN ONE DIMENSION

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.18.

Here's a simple example of calculating the fields due to a moving point charge. Suppose we have a charge constrained to move on the x axis in the $+x$ direction, so that $\mathbf{v}=v\hat{\mathbf{x}}$.

The fields are

$$\mathbf{E}(\mathbf{r},t) = \frac{q\mathfrak{r}}{4\pi\varepsilon_0(\boldsymbol{\mathfrak{r}}\cdot\mathbf{u})^3}\left[\left(c^2-v^2\right)\mathbf{u}+\boldsymbol{\mathfrak{r}}\times(\mathbf{u}\times\mathbf{a})\right] \tag{1}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\boldsymbol{\mathfrak{r}}}\times\mathbf{E}(\mathbf{r},t) \tag{2}$$

where

$$\boldsymbol{\mathfrak{r}} \equiv \mathbf{r}-\mathbf{w}(t_r) \tag{3}$$

$$\mathbf{u} \equiv c\hat{\boldsymbol{\mathfrak{r}}}-\mathbf{v} \tag{4}$$

and $\mathbf{w}(t_r)$ is the particle's position at the retarded time. If the observer is to the right of the particle then

$$\boldsymbol{\mathfrak{r}} = +\mathfrak{r}\hat{\mathbf{x}} \tag{5}$$

$$\mathbf{u} = (c-v)\hat{\mathbf{x}} \tag{6}$$

Since the motion is constrained to the x axis, any velocity and acceleration must be parallel, so $\mathbf{u}\times\mathbf{a}=0$ and we have

$$\mathbf{E} = \frac{q\mathfrak{r}}{4\pi\varepsilon_0(\boldsymbol{\mathfrak{r}}\cdot\mathbf{u})^3}\left(c^2-v^2\right)\mathbf{u} \tag{7}$$

$$= \frac{q\left(c^2-v^2\right)}{4\pi\varepsilon_0(c-v)^3\mathfrak{r}^2}(c-v)\hat{\mathbf{x}} \tag{8}$$

$$= \frac{q(c+v)}{4\pi\varepsilon_0(c-v)\mathfrak{r}^2}\hat{\mathbf{x}} \tag{9}$$

Because $\hat{\boldsymbol{\mathfrak{r}}}$ is parallel to $\mathbf{E}$, $\mathbf{B}=0$ from 2.

If the observer is to the left of the charge, then

$$\boldsymbol{\mathfrak{r}} = -\mathfrak{r}\hat{\mathbf{x}} \tag{10}$$

$$\mathbf{u} = -(c+v)\hat{\mathbf{x}} \tag{11}$$

$$\mathbf{E} = \frac{q\mathfrak{r}}{4\pi\varepsilon_0(\boldsymbol{\mathfrak{r}}\cdot\mathbf{u})^3}\left(c^2-v^2\right)\mathbf{u} \tag{12}$$

$$= -\frac{q\left(c^2-v^2\right)}{4\pi\varepsilon_0(c+v)^3\mathfrak{r}^2}(c+v)\hat{\mathbf{x}} \tag{13}$$

$$= -\frac{q(c-v)}{4\pi\varepsilon_0(c+v)\mathfrak{r}^2}\hat{\mathbf{x}} \tag{14}$$

$$\mathbf{B} = 0 \tag{15}$$