

## RETARDED POTENTIALS FOR A SINUSOIDAL CURRENT LOOP

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.21.

Here's another example of calculating retarded potentials

$$(0.1) \quad V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{d} d^3\mathbf{r}'$$

$$(0.2) \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{d} d^3\mathbf{r}'$$

We have a circular loop of radius  $a$  with a charge distribution (at  $t = 0$ ) of  $\lambda_0 \sin \frac{\theta}{2}$ . The loop is then spun at angular velocity  $\omega$  and we wish to find the potentials at the centre of the loop.

In this case, the distance from the observation point ( $\mathbf{r} = 0$ ) to the retarded position of the charge is always the same ( $d = a$ ) so the potentials become

$$(0.3) \quad V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0 a} \int \rho(\mathbf{r}', t_r) d^3\mathbf{r}'$$

$$(0.4) \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi a} \int \mathbf{J}(\mathbf{r}', t_r) d^3\mathbf{r}'$$

The scalar potential's integral is thus just the total charge on the loop, which doesn't depend on time so we have

$$(0.5) \quad V(\mathbf{r}, t) = \frac{\lambda_0 a}{4\pi\epsilon_0 a} \int_0^{2\pi} \sin \frac{\theta}{2} d\theta$$

$$(0.6) \quad = \frac{\lambda_0}{\pi\epsilon_0}$$

For the vector potential we need  $\mathbf{J}(\mathbf{r}', t_r)$ . To get this, we use the following argument. At  $t = 0$ , the charge element at angle  $\theta$  is  $\lambda_0 a \sin \frac{\theta}{2}$ . At time  $t$ , this element has moved through angle  $\omega t$  so it's now at an angle of  $\theta + \omega t$ , so the linear charge density as a function of time is

$$(0.7) \quad \lambda(t) = \lambda_0 a \sin \frac{\theta}{2} [\cos(\theta + \omega t) \hat{\mathbf{x}} + \sin(\theta + \omega t) \hat{\mathbf{y}}]$$

The linear current is therefore

$$(0.8) \quad \mathbf{I}(\theta, t) = \dot{\lambda} = \lambda_0 a \omega \sin \frac{\theta}{2} [-\sin(\theta + \omega t) \hat{\mathbf{x}} + \cos(\theta + \omega t) \hat{\mathbf{y}}]$$

The integral of the current is

(0.9)

$$\int \mathbf{J}(\mathbf{r}', t_r) d^3 \mathbf{r}' = \int_0^{2\pi} \mathbf{I}(\theta, t_r)$$

$$(0.10) \quad = \lambda_0 a \omega \int_0^{2\pi} \left[ -\sin \frac{\theta}{2} \sin(\theta + \omega t_r) \hat{\mathbf{x}} + \sin \frac{\theta}{2} \cos(\theta + \omega t_r) \hat{\mathbf{y}} \right] d\theta$$

$$(0.11) \quad = \frac{4}{3} \lambda_0 a \omega \left[ \sin \left( \omega \left( t - \frac{a}{c} \right) \right) \hat{\mathbf{x}} - \cos \left( \omega \left( t - \frac{a}{c} \right) \right) \hat{\mathbf{y}} \right]$$

[The integrals can be done using the trigonometric addition formulas for  $\cos(x+y)$  and  $\sin(x+y)$ .]

The vector potential is therefore

$$(0.12) \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0 \lambda_0 a \omega}{3\pi} \left[ \sin \left( \omega \left( t - \frac{a}{c} \right) \right) \hat{\mathbf{x}} - \cos \left( \omega \left( t - \frac{a}{c} \right) \right) \hat{\mathbf{y}} \right]$$