

RELATIONS AMONG CHARGE, CURRENT, POTENTIALS AND FIELDS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.22.

With a complete theory of electrodynamics, it's useful to summarize the relations between the sources of the fields (charge density ρ and current density \mathbf{J}), the potentials V and \mathbf{A} and the fields \mathbf{E} and \mathbf{B} .

Starting with ρ and \mathbf{J} , we can calculate the retarded potentials

$$(1) \quad V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{d} d^3\mathbf{r}'$$

$$(2) \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{d} d^3\mathbf{r}'$$

Or we can get the fields from Jefimenko's equations for the electric and magnetic fields.

$$(3) \quad \mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \left(\frac{\rho(\mathbf{r}', t_r)}{r^2} \hat{\mathbf{r}} + \frac{\dot{\rho}(\mathbf{r}', t_r)}{cr} \hat{\mathbf{r}} - \frac{\mathbf{J}(\mathbf{r}', t_r)}{c^2 r} \right) d^3\mathbf{r}'$$

$$(4) \quad \mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \left[\frac{\mathbf{J}(\mathbf{r}', t_r) \times \hat{\mathbf{r}}}{cr} + \mathbf{J}(\mathbf{r}', t_r) \times \frac{\hat{\mathbf{r}}}{r^2} \right] d^3\mathbf{r}'$$

Inverting the procedure, we can get the sources from the potentials, although we need to know the gauge we're using. In the Lorenz gauge we have

$$(5) \quad \square^2 V = -\frac{\rho}{\epsilon_0}$$

$$(6) \quad \square^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

From the fields, we can use Maxwell's equations

$$(7) \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$(8) \quad \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

Finally, from the potentials we can get the fields

$$(9) \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$(10) \quad \mathbf{B} = \nabla \times \mathbf{A}$$