RELATIONS AMONG CHARGE, CURRENT, POTENTIALS AND FIELDS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.22.

With a complete theory of electrodynamics, it's useful to summarize the relations between the sources of the fields (charge density ρ and current density $\mathbf{J}$), the potentials V and $\mathbf{A}$ and the fields $\mathbf{E}$ and $\mathbf{B}$.

Starting with ρ and $\mathbf{J}$, we can calculate the retarded potentials

$$V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\mathbf{r}',t_r)}{d}d^3\mathbf{r}' \tag{0.1}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{d}d^3\mathbf{r}' \tag{0.2}$$

Or we can get the fields from Jefimenko's equations for the electric and magnetic fields.

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left(\frac{\rho(\mathbf{r}',t_r)}{\mathfrak{r}^2}\hat{\mathfrak{r}}+\frac{\dot{\rho}(\mathbf{r}',t_r)}{c\mathfrak{r}}\hat{\mathfrak{r}}-\frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c^2\mathfrak{r}}\right)d^3\mathbf{r}' \tag{0.3}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)\times\hat{\mathfrak{r}}}{cr}+\mathbf{J}(\mathbf{r}',t_r)\times\frac{\hat{\mathfrak{r}}}{r^2}\right]d^3\mathbf{r}' \tag{0.4}$$

Inverting the procedure, we can get the sources from the potentials, although we need to know the gauge we're using. In the Lorenz gauge we have

$$\Box^2 V = -\frac{\rho}{\varepsilon_0} \tag{0.5}$$

$$\Box^2\mathbf{A} = -\mu_0\mathbf{J} \tag{0.6}$$

From the fields, we can use Maxwell's equations

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{0.7}$$

$$\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J} \tag{0.8}$$

Finally, from the potentials we can get the fields

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \tag{0.9}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{0.10}$$