

## RELATIONS AMONG CHARGE, CURRENT, POTENTIALS AND FIELDS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.22.

With a complete theory of electrodynamics, it's useful to summarize the relations between the sources of the fields (charge density  $\rho$  and current density  $\mathbf{J}$ ), the potentials  $V$  and  $\mathbf{A}$  and the fields  $\mathbf{E}$  and  $\mathbf{B}$ .

Starting with  $\rho$  and  $\mathbf{J}$ , we can calculate the retarded potentials

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{d} d^3\mathbf{r}' \quad (1)$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{d} d^3\mathbf{r}' \quad (2)$$

Or we can get the fields from Jefimenko's equations for the electric and magnetic fields.

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \left( \frac{\rho(\mathbf{r}', t_r)}{r^2} \hat{\mathbf{r}} + \frac{\dot{\rho}(\mathbf{r}', t_r)}{cr} \hat{\mathbf{r}} - \frac{\mathbf{J}(\mathbf{r}', t_r)}{c^2 r} \right) d^3\mathbf{r}' \quad (3)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \left[ \frac{\mathbf{J}(\mathbf{r}', t_r) \times \hat{\mathbf{r}}}{cr} + \mathbf{J}(\mathbf{r}', t_r) \times \frac{\hat{\mathbf{r}}}{r^2} \right] d^3\mathbf{r}' \quad (4)$$

Inverting the procedure, we can get the sources from the potentials, although we need to know the gauge we're using. In the Lorenz gauge we have

$$\square^2 V = -\frac{\rho}{\epsilon_0} \quad (5)$$

$$\square^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad (6)$$

From the fields, we can use Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (7)$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad (8)$$

Finally, from the potentials we can get the fields

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad (9)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (10)$$