POINT CHARGE MOVING AT CONSTANT VELOCITY SATISFIES LORENZ GAUGE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 10.23.

Griffiths shows in his example 10.3 that the Liénard-Wiechert potentials for a point charge q moving at constant velocity $\mathbf{v}$ that passes through the origin at time $t=0$ are

$$V(\mathbf{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qc}{\sqrt{\left(c^2t-\mathbf{r}\cdot\mathbf{v}\right)^2+\left(c^2-v^2\right)\left(r^2-c^2t^2\right)}} \tag{0.1}$$

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{\sqrt{\left(c^2t-\mathbf{r}\cdot\mathbf{v}\right)^2+\left(c^2-v^2\right)\left(r^2-c^2t^2\right)}} \tag{0.2}$$

These potentials were derived from the general retarded potential formulas that in turn satisfy the Lorenz gauge condition

$$\nabla\cdot\mathbf{A}=-\frac{1}{c^2}\frac{\partial V}{\partial t} \tag{0.3}$$

It is possible to show explicitly that these two potentials satisfy the Lorenz gauge condition, although to do so by hand is straightforward, but rather tedious. It's easier to let Maple handle the derivatives, and we find

$$\nabla\cdot\mathbf{A}=\frac{qc\left(v^2t-\mathbf{r}\cdot\mathbf{v}\right)}{4\pi\varepsilon_0\left(\left(c^2t-\mathbf{r}\cdot\mathbf{v}\right)^2+\left(c^2-v^2\right)\left(r^2-c^2t^2\right)\right)^{3/2}} \tag{0.4}$$

$$\frac{\partial V}{\partial t}=-\frac{qc^3\left(v^2t-\mathbf{r}\cdot\mathbf{v}\right)}{4\pi\varepsilon_0\left(\left(c^2t-\mathbf{r}\cdot\mathbf{v}\right)^2+\left(c^2-v^2\right)\left(r^2-c^2t^2\right)\right)^{3/2}} \tag{0.5}$$

so the condition is satisfied.