

FIELDS AND RADIATED POWER FROM AN OSCILLATING ELECTRIC DIPOLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 2

The potentials for an oscillating dipole at a large distance from the dipole are

$$(0.1) \quad V(r, \theta, t) = -\frac{p_0 \omega \cos \theta}{4\pi \epsilon_0 r c} \sin\left(\omega\left(t - \frac{r}{c}\right)\right)$$

$$(0.2) \quad \mathbf{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \hat{\mathbf{z}} \sin\left(\omega\left(t - \frac{r}{c}\right)\right)$$

These formulas apply in the special case where the dipole axis is the z axis, so that the dipole moment is

$$(0.3) \quad \mathbf{p} = p_0 \cos(\omega t) \hat{\mathbf{z}}$$

We can rewrite these formulas for a dipole pointing in any direction by noting that

$$(0.4) \quad p_0 \cos \theta = \mathbf{p}_0 \cdot \hat{\mathbf{r}}$$

so

$$(0.5) \quad V(r, \theta, t) = -\frac{\omega}{4\pi \epsilon_0 r c} (\mathbf{p}_0 \cdot \hat{\mathbf{r}}) \sin\left(\omega\left(t - \frac{r}{c}\right)\right)$$

$$(0.6) \quad \mathbf{A}(r, \theta, t) = -\frac{\mu_0 \omega}{4\pi r} \mathbf{p}_0 \sin\left(\omega\left(t - \frac{r}{c}\right)\right)$$

The fields can be calculated from the potentials using straightforward differentiation. Griffiths shows the details in his section 11.1.2. After assuming that $r \gg \frac{c}{\omega}$ (equivalent to assuming that the observation point is much greater than the wavelength of the radiation) we get from 0.1 and 0.2:

$$(0.7) \quad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$(0.8) \quad = -\frac{\mu_0 p_0 \omega^2 \sin \theta}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \hat{\theta}$$

$$(0.9) \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$(0.10) \quad = -\frac{\mu_0 p_0 \omega^2 \sin \theta}{4\pi c r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \hat{\phi}$$

Note that \mathbf{E} and \mathbf{B} are perpendicular and in phase, and that $E/B = c$ just as with plane waves in vacuum. We can write these equations for general dipole directions by noting that

$$(0.11) \quad \hat{\mathbf{z}} \times \hat{\mathbf{r}} = \sin \theta \hat{\phi}$$

$$(0.12) \quad \hat{\phi} \times \hat{\mathbf{r}} = \hat{\theta}$$

$$(0.13) \quad (\hat{\mathbf{z}} \times \hat{\mathbf{r}}) \times \hat{\mathbf{r}} = \sin \theta \hat{\theta}$$

Therefore

$$(0.14) \quad \mathbf{E} = -\frac{\mu_0 \omega^2}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) (\hat{\mathbf{p}}_0 \times \hat{\mathbf{r}}) \times \hat{\mathbf{r}}$$

$$(0.15) \quad \mathbf{B} = -\frac{\mu_0 \omega^2}{4\pi r c} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) (\hat{\mathbf{p}}_0 \times \hat{\mathbf{r}})$$

The energy radiated per unit area per unit time is given by the Poynting vector:

$$(0.16) \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

$$(0.17) \quad = \frac{\mu_0}{c} \left[\frac{p_0 \omega^2 \sin \theta}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \right]^2 \hat{\mathbf{r}}$$

For a general dipole direction, this is

$$(0.18) \quad \mathbf{S} = \frac{\mu_0}{c} \left[\frac{\omega^2 |\mathbf{p}_0 \times \hat{\mathbf{r}}|}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \right]^2 \hat{\mathbf{r}}$$

The intensity is the average of \mathbf{S} over a single time cycle (that is, over a time $2\pi/\omega$). The average of $\cos^2 x$ over a single cycle is $\frac{1}{2}$, so

$$(0.19) \quad \langle \mathbf{S} \rangle = \frac{\mu_0}{2c} \left[\frac{\omega^2 p_0 \sin \theta}{4\pi r} \right]^2 \hat{\mathbf{r}}$$

or in direction-independent form

$$(0.20) \quad \langle \mathbf{S} \rangle = \frac{\mu_0}{2c} \left[\frac{\omega^2 |\mathbf{p}_0 \times \hat{\mathbf{r}}|}{4\pi r} \right]^2 \hat{\mathbf{r}}$$

There is no radiation along the dipole's axis, and the maximum radiation occurs perpendicular to the axis.

The average total power radiated is the surface integral of $\langle \mathbf{S} \rangle$ over a sphere of radius r , so we get from 0.19

$$(0.21) \quad \langle P \rangle = \int \frac{\mu_0}{2c} \left[\frac{\omega^2 p_0 \sin \theta}{4\pi r} \right]^2 r^2 \sin \theta \hat{\mathbf{r}} \cdot d\mathbf{a}$$

$$(0.22) \quad = \frac{\mu_0 \omega^4 p_0^2}{32\pi^2 c} \int_0^\pi \int_0^{2\pi} \sin^3 \theta d\phi d\theta$$

$$(0.23) \quad = \frac{\mu_0 \omega^4 p_0^2}{12\pi c}$$

The result is independent of distance r from the dipole, so we see that this power remains constant out to infinity.

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