FIELDS AND RADIATED POWER FROM AN OSCILLATING ELECTRIC DIPOLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 2

The potentials for an oscillating dipole at a large distance from the dipole are

$$V(r,\theta,t) = -\frac{p_0\omega\cos\theta}{4\pi\varepsilon_0 rc}\sin\left(\omega\left(t-\frac{r}{c}\right)\right) \tag{1}$$

$$\mathbf{A}(r,\theta,t) = -\frac{\mu_0 p_0\omega}{4\pi r}\hat{\mathbf{z}}\sin\left(\omega\left(t-\frac{r}{c}\right)\right) \tag{2}$$

These formulas apply in the special case where the dipole axis is the z axis, so that the dipole moment is

$$\mathbf{p} = p_0\cos(\omega t)\hat{\mathbf{z}} \tag{3}$$

We can rewrite these formulas for a dipole pointing in any direction by noting that

$$p_0\cos\theta = \mathbf{p}_0\cdot\hat{\mathbf{r}} \tag{4}$$

so

$$V(r,\theta,t) = -\frac{\omega}{4\pi\varepsilon_0 rc}(\mathbf{p}_0\cdot\hat{\mathbf{r}})\sin\left(\omega\left(t-\frac{r}{c}\right)\right) \tag{5}$$

$$\mathbf{A}(r,\theta,t) = -\frac{\mu_0\omega}{4\pi r}\mathbf{p}_0\sin\left(\omega\left(t-\frac{r}{c}\right)\right) \tag{6}$$

The fields can be calculated from the potentials using straightforward differentiation. Griffiths shows the details in his section 11.1.2. After assuming that $r \gg \frac{c}{\omega}$ (equivalent to assuming that the observation point is much greater than the wavelength of the radiation) we get from 1 and 2:

$$\begin{aligned}
\mathbf{E} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} && (7)\\
&= -\frac{\mu_0 p_0 \omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\theta} && (8)\\
\mathbf{B} &= \nabla\times\mathbf{A} && (9)\\
&= -\frac{\mu_0 p_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\phi} && (10)
\end{aligned}$$

Note that $\mathbf{E}$ and $\mathbf{B}$ are perpendicular and in phase, and that $E/B = c$ just as with plane waves in vacuum. We can write these equations for general dipole directions by noting that

$$\begin{aligned}
\hat{\mathbf{z}}\times\hat{\mathbf{r}} &= \sin\theta\hat{\phi} && (11)\\
\hat{\phi}\times\hat{\mathbf{r}} &= \hat{\theta} && (12)\\
(\hat{\mathbf{z}}\times\hat{\mathbf{r}})\times\hat{\mathbf{r}} &= \sin\theta\hat{\theta} && (13)
\end{aligned}$$

Therefore

$$\begin{aligned}
\mathbf{E} &= -\frac{\mu_0\omega^2}{4\pi r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)(\hat{\mathbf{p}}_0\times\hat{\mathbf{r}})\times\hat{\mathbf{r}} && (14)\\
\mathbf{B} &= -\frac{\mu_0\omega^2}{4\pi rc}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)(\hat{\mathbf{p}}_0\times\hat{\mathbf{r}}) && (15)
\end{aligned}$$

The energy radiated per unit area per unit time is given by the Poynting vector:

$$\begin{aligned}
\mathbf{S} &= \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} && (16)\\
&= \frac{\mu_0}{c}\left[\frac{p_0\omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\right]^2\hat{\mathbf{r}} && (17)
\end{aligned}$$

For a general dipole direction, this is

$$\mathbf{S} = \frac{\mu_0}{c}\left[\frac{\omega^2}{4\pi}\frac{|\mathbf{p}_0\times\hat{\mathbf{r}}|}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\right]^2\hat{\mathbf{r}} \tag{18}$$

The intensity is the average of $\mathbf{S}$ over a single time cycle (that is, over a time $2\pi/\omega$). The average of $\cos^2 x$ over a single cycle is $\frac{1}{2}$, so

$$\langle\mathbf{S}\rangle = \frac{\mu_0}{2c}\left[\frac{\omega^2}{4\pi}\frac{p_0\sin\theta}{r}\right]^2\hat{\mathbf{r}} \tag{19}$$

or in direction-independent form

$$\langle\mathbf{S}\rangle=\frac{\mu_0}{2c}\left[\frac{\omega^2}{4\pi}\frac{|\mathbf{p}_0\times\hat{\mathbf{r}}|}{r}\right]^2\hat{\mathbf{r}} \tag{20}$$

There is no radiation along the dipole's axis, and the maximum radiation occurs perpendicular to the axis.

The average total power radiated is the surface integral of $\langle\mathbf{S}\rangle$ over a sphere of radius r, so we get from 19

$$\langle P\rangle = \int\frac{\mu_0}{2c}\left[\frac{\omega^2}{4\pi}\frac{p_0\sin\theta}{r}\right]^2 r^2\sin\theta\hat{\mathbf{r}}\cdot d\mathbf{a} \tag{21}$$

$$= \frac{\mu_0\omega^4p_0^2}{32\pi^2c}\int_0^\pi\int_0^{2\pi}\sin^3\theta d\phi d\theta \tag{22}$$

$$= \frac{\mu_0\omega^4p_0^2}{12\pi c} \tag{23}$$

The result is independent of distance r from the dipole, so we see that this power remains constant out to infinity.

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