

## FIELDS AND RADIATED POWER FROM AN OSCILLATING ELECTRIC DIPOLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 2

The potentials for an oscillating dipole at a large distance from the dipole are

$$V(r, \theta, t) = -\frac{p_0 \omega \cos \theta}{4\pi \epsilon_0 r c} \sin\left(\omega\left(t - \frac{r}{c}\right)\right) \quad (1)$$

$$\mathbf{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \hat{\mathbf{z}} \sin\left(\omega\left(t - \frac{r}{c}\right)\right) \quad (2)$$

These formulas apply in the special case where the dipole axis is the  $z$  axis, so that the dipole moment is

$$\mathbf{p} = p_0 \cos(\omega t) \hat{\mathbf{z}} \quad (3)$$

We can rewrite these formulas for a dipole pointing in any direction by noting that

$$p_0 \cos \theta = \mathbf{p}_0 \cdot \hat{\mathbf{r}} \quad (4)$$

so

$$V(r, \theta, t) = -\frac{\omega}{4\pi \epsilon_0 r c} (\mathbf{p}_0 \cdot \hat{\mathbf{r}}) \sin\left(\omega\left(t - \frac{r}{c}\right)\right) \quad (5)$$

$$\mathbf{A}(r, \theta, t) = -\frac{\mu_0 \omega}{4\pi r} \mathbf{p}_0 \sin\left(\omega\left(t - \frac{r}{c}\right)\right) \quad (6)$$

The fields can be calculated from the potentials using straightforward differentiation. Griffiths shows the details in his section 11.1.2. After assuming that  $r \gg \frac{c}{\omega}$  (equivalent to assuming that the observation point is much greater than the wavelength of the radiation) we get from 1 and 2:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad (7)$$

$$= -\frac{\mu_0 p_0 \omega^2 \sin \theta}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \hat{\theta} \quad (8)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (9)$$

$$= -\frac{\mu_0 p_0 \omega^2 \sin \theta}{4\pi c r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \hat{\phi} \quad (10)$$

Note that  $\mathbf{E}$  and  $\mathbf{B}$  are perpendicular and in phase, and that  $E/B = c$  just as with plane waves in vacuum. We can write these equations for general dipole directions by noting that

$$\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \sin \theta \hat{\phi} \quad (11)$$

$$\hat{\phi} \times \hat{\mathbf{r}} = \hat{\theta} \quad (12)$$

$$(\hat{\mathbf{z}} \times \hat{\mathbf{r}}) \times \hat{\mathbf{r}} = \sin \theta \hat{\theta} \quad (13)$$

Therefore

$$\mathbf{E} = -\frac{\mu_0 \omega^2}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) (\hat{\mathbf{p}}_0 \times \hat{\mathbf{r}}) \times \hat{\mathbf{r}} \quad (14)$$

$$\mathbf{B} = -\frac{\mu_0 \omega^2}{4\pi r c} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) (\hat{\mathbf{p}}_0 \times \hat{\mathbf{r}}) \quad (15)$$

The energy radiated per unit area per unit time is given by the Poynting vector:

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad (16)$$

$$= \frac{\mu_0}{c} \left[ \frac{p_0 \omega^2 \sin \theta}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \right]^2 \hat{\mathbf{r}} \quad (17)$$

For a general dipole direction, this is

$$\mathbf{S} = \frac{\mu_0}{c} \left[ \frac{\omega^2 |\mathbf{p}_0 \times \hat{\mathbf{r}}|}{4\pi r} \cos\left(\omega\left(t - \frac{r}{c}\right)\right) \right]^2 \hat{\mathbf{r}} \quad (18)$$

The intensity is the average of  $\mathbf{S}$  over a single time cycle (that is, over a time  $2\pi/\omega$ ). The average of  $\cos^2 x$  over a single cycle is  $\frac{1}{2}$ , so

$$\langle \mathbf{S} \rangle = \frac{\mu_0}{2c} \left[ \frac{\omega^2 p_0 \sin \theta}{4\pi r} \right]^2 \hat{\mathbf{r}} \quad (19)$$

or in direction-independent form

$$\langle \mathbf{S} \rangle = \frac{\mu_0}{2c} \left[ \frac{\omega^2}{4\pi} \frac{|\mathbf{p}_0 \times \hat{\mathbf{r}}|}{r} \right]^2 \hat{\mathbf{r}} \quad (20)$$

There is no radiation along the dipole's axis, and the maximum radiation occurs perpendicular to the axis.

The average total power radiated is the surface integral of  $\langle \mathbf{S} \rangle$  over a sphere of radius  $r$ , so we get from 19

$$\langle P \rangle = \int \frac{\mu_0}{2c} \left[ \frac{\omega^2 p_0 \sin \theta}{4\pi r} \right]^2 r^2 \sin \theta \hat{\mathbf{r}} \cdot d\mathbf{a} \quad (21)$$

$$= \frac{\mu_0 \omega^4 p_0^2}{32\pi^2 c} \int_0^\pi \int_0^{2\pi} \sin^3 \theta d\phi d\theta \quad (22)$$

$$= \frac{\mu_0 \omega^4 p_0^2}{12\pi c} \quad (23)$$

The result is independent of distance  $r$  from the dipole, so we see that this power remains constant out to infinity.

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