

RADIATION RESISTANCE OF AN OSCILLATING MAGNETIC DIPOLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 6

We can find the radiation resistance of an oscillating magnetic dipole produced by an AC current in a circular wire loop of radius b in the same way as for an oscillating electric dipole. The average power radiated by the magnetic dipole is the integral of the intensity over a sphere, so we have

$$\begin{aligned} (1) \quad \langle P \rangle &= \int \langle \mathbf{S} \rangle \cdot d\mathbf{a} \\ (2) \quad &= \int \frac{\mu_0 m_0^2 \omega^4 \sin^2 \theta}{32\pi^2 c^3 r^2} \hat{\mathbf{r}} \cdot d\mathbf{a} \\ (3) \quad &= \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} 2\pi \int_0^\pi \frac{\sin^2 \theta}{r^2} r^2 \sin \theta d\theta \\ (4) \quad &= \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \end{aligned}$$

The resistance R required in the wire loop to generate the same power loss through heat is given by $P = \langle I^2 \rangle R$ where the current is

$$(5) \quad I(t) = I_0 \cos \omega t$$

with the maximum current I_0 given in terms of the maximum magnetic moment m_0 :

$$(6) \quad I_0 = \frac{m_0}{\pi b^2}$$

Since the average of $\cos^2 x$ over a complete cycle is $\frac{1}{2}$ we get

$$\begin{aligned}
 (7) \quad \langle I^2 \rangle R &= \frac{m_0^2}{2\pi^2 b^4} R \\
 (8) &= \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \\
 (9) \quad R &= \frac{\mu_0 \pi \omega^4 b^4}{6c^3} \\
 (10) &= \frac{8\mu_0 \pi^5 c b^4}{3 \lambda^4} \\
 (11) &= 3.08 \times 10^5 \frac{b^4}{\lambda^4} \Omega
 \end{aligned}$$

where in the fourth line we used $\omega/c = 2\pi/\lambda$.

To compare this to the electric dipole's radiation resistance, which is given in terms of the length l of the wire joining the two charges as:

$$(12) \quad R_e = 787 \frac{l^2}{\lambda^2} \Omega$$

we can take $l = 2\pi b$ so the lengths of wire in the two cases are the same. Then

$$\begin{aligned}
 (13) \quad \frac{R_e}{R_m} &= \frac{787 (2\pi)^2 \lambda^2}{3.08 \times 10^5 b^2} \\
 (14) &= 0.1 \frac{\lambda^2}{b^2}
 \end{aligned}$$

Since we're assuming that $b \ll \lambda$, $R_e \gg R_m$.