RADIATION RESISTANCE OF AN OSCILLATING MAGNETIC DIPOLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 6

We can find the radiation resistance of an oscillating magnetic dipole produced by an AC current in a circular wire loop of radius b in the same way as for an oscillating electric dipole. The average power radiated by the magnetic dipole is the integral of the intensity over a sphere, so we have

$$\begin{aligned}
\langle P\rangle &= \int\langle\mathbf{S}\rangle\cdot d\mathbf{a} && (1)\\
&= \int\frac{\mu_0 m_0^2\omega^4\sin^2\theta}{32\pi^2c^3r^2}\hat{\mathbf{r}}\cdot d\mathbf{a} && (2)\\
&= \frac{\mu_0 m_0^2\omega^4}{32\pi^2c^3}2\pi\int_0^\pi\frac{\sin^2\theta}{r^2}r^2\sin\theta d\theta && (3)\\
&= \frac{\mu_0 m_0^2\omega^4}{12\pi c^3} && (4)
\end{aligned}$$

The resistance R required in the wire loop to generate the same power loss through heat is given by $P=\left\langle I^2\right\rangle R$ where the current is

$$I(t)=I_0\cos\omega t \tag{5}$$

with the maximum current I_0 given in terms of the maximum magnetic moment m_0:

$$I_0=\frac{m_0}{\pi b^2} \tag{6}$$

Since the average of $\cos^2 x$ over a complete cycle is $\frac{1}{2}$ we get

$$\begin{aligned}
\langle I^2 \rangle R &= \frac{m_0^2}{2\pi^2 b^4} R && (7)\\
&= \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} && (8)\\
R &= \frac{\mu_0 \pi \omega^4 b^4}{6c^3} && (9)\\
&= \frac{8\mu_0 \pi^5 c}{3} \frac{b^4}{\lambda^4} && (10)\\
&= 3.08 \times 10^5 \frac{b^4}{\lambda^4}\ \Omega && (11)
\end{aligned}$$

where in the fourth line we used $\omega/c = 2\pi/\lambda$.

To compare this to the electric dipole's radiation resistance, which is given in terms of the length l of the wire joining the two charges as:

$$R_e = 787 \frac{l^2}{\lambda^2}\ \Omega \tag{12}$$

we can take $l = 2\pi b$ so the lengths of wire in the two cases are the same. Then

$$\begin{aligned}
\frac{R_e}{R_m} &= \frac{787\,(2\pi)^2}{3.08 \times 10^5} \frac{\lambda^2}{b^2} && (13)\\
&= 0.1 \frac{\lambda^2}{b^2} && (14)
\end{aligned}$$

Since we're assuming that $b \ll \lambda$, $R_e \gg R_m$.