RADIATION FROM A MAGNETIC DIPOLE COMPOSED OF MONOPOLES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 7

We've seen that the fields produced by an oscillating magnetic dipole are

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \tag{1}$$

$$= \frac{\mu_0 m_0 \sin\theta}{4\pi r}\left\{\frac{\omega^2}{c}\cos\left[\omega\left(t-r/c\right)\right]+\frac{\omega}{r}\sin\left[\omega\left(t-r/c\right)\right]\right\}\hat{\phi} \tag{2}$$

$$\mathbf{B} = \nabla\times\mathbf{A} \tag{3}$$

$$= \frac{\mu_0 m_0 \cos\theta}{2\pi r^2}\left[\frac{1}{r}\cos\left[\omega\left(t-r/c\right)\right]-\frac{\omega}{c}\sin\left[\omega\left(t-r/c\right)\right]\right]\hat{\mathbf{r}}+ \frac{\mu_0 m_0 \sin\theta}{4\pi r}\left[\left(\frac{1}{r^2}-\frac{\omega^2}{c^2}\right)\cos\left[\omega\left(t-r/c\right)\right]-\frac{\omega}{rc}\sin\left[\omega\left(t-r/c\right)\right]\right]\hat{\theta} \tag{4}$$

By making the approximation that the observation distance r is much larger than the wavelength of radiation, so that $r \gg c/\omega$, these formulas simplify to

$$\mathbf{E} = \frac{\mu_0 m_0 \omega^2 \sin\theta}{4\pi c r}\hat{\phi}\cos\left[\omega\left(t-r/c\right)\right] \tag{5}$$

$$\mathbf{B} = -\frac{\mu_0 m_0 \omega^2 \sin\theta}{4\pi c^2 r}\hat{\theta}\cos\left[\omega\left(t-r/c\right)\right] \tag{6}$$

Now let's return to the fantasy world where magnetic monopoles exist, so that another way we can create a magnetic dipole is to connect two magnetic charges by a wire and then drive charge back and forth between the ends of the wire, in the same way that we did for the electric dipole. Earlier, we've seen that if we include magnetic charge in Maxwell's equations, the duality transformation produces fields that still satisfy Maxwell's equations:

$$\mathbf{E}' = \mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha \tag{7}$$
$$c\mathbf{B}' = c\mathbf{B}\cos\alpha - \mathbf{E}\sin\alpha \tag{8}$$
$$cq_e' = cq_e\cos\alpha + q_m\sin\alpha \tag{9}$$
$$q_m' = q_m\cos\alpha - cq_e\sin\alpha \tag{10}$$

where α is a rotation angle in $\mathbf{E}-\mathbf{B}$ space. If we start with the fields generated by an oscillating electric dipole and then choose $\alpha = \frac{\pi}{2}$ so that we convert all electric charge into magnetic charge, we can generate the fields that would be produced by an oscillating magnetic dipole constructed using magnetic charge as described above. The original fields for the electric dipole are

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \tag{11}$$
$$= -\frac{\mu_0 p_0 \omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\theta} \tag{12}$$
$$\mathbf{B} = \nabla\times\mathbf{A} \tag{13}$$
$$= -\frac{\mu_0 p_0 \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\phi} \tag{14}$$

The required transformations with $\alpha = \frac{\pi}{2}$ are

$$\mathbf{E}' = c\mathbf{B} \tag{15}$$
$$c\mathbf{B}' = -\mathbf{E} \tag{16}$$
$$cq_e' = q_m \tag{17}$$
$$q_m' = -cq_e \tag{18}$$

As the electric dipole moment $p_0 = q_e l$ is the product of an electric charge q_e and the length l of the wire joining the two charges, it transforms in the same way as q_e so we have, if we take the magnetic moment to be $m_0 = q_m' l$:

$$m_0 = -cp_0 \tag{19}$$

Applying these transformations to 12 and 14 we get

$$\mathbf{E}' = -c\frac{\mu_0(-m_0/c)\,\omega^2}{4\pi c}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\phi} \tag{20}$$

$$= \frac{\mu_0 m_0 \omega^2 \sin\theta}{4\pi c r}\hat{\phi}\cos\left[\omega\left(t-r/c\right)\right] \tag{21}$$

$$\mathbf{B}' = -\left(-\frac{1}{c}\right)\frac{\mu_0(-m_0/c)\,\omega^2}{4\pi}\frac{\sin\theta}{r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat{\theta} \tag{22}$$

$$= -\frac{\mu_0 m_0 \omega^2 \sin\theta}{4\pi c^2 r}\hat{\theta}\cos\left[\omega\left(t-r/c\right)\right] \tag{23}$$

These fields are the same as 5 and 6 that we got from the current loop. Thus we can't tell whether magnetic dipole radiation is coming from a current loop or from magnetic monopoles.