POWER RADIATED BY A SPINNING RING OF CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 9.

Here's a generalization of the rotating dipole problem we did earlier. This time we have a circular ring with radius b with a linear charge distribution, at $t=0$, of

$$\lambda=\lambda_0\sin\phi \tag{1}$$

where ϕ is the azimuthal angle. The disk is set spinning with an angular velocity of ω.

Because $\sin(\phi+\pi)=-\sin\phi$, this disk is essentially a collection of dipoles with charges $\pm\lambda b\,d\phi$ separated by distance $2b$. Therefore, at time t, the dipole moment is

$$\mathbf{p}(t)=2b^2\lambda_0\int_0^\pi\sin\phi\left[\cos(\omega t+\phi)\hat{\mathbf{x}}+\sin(\omega t+\phi)\hat{\mathbf{y}}\right]d\phi \tag{2}$$

$$=2b^2\lambda_0\frac{\pi}{2}\left(-\sin\omega t\hat{\mathbf{x}}+\cos\omega t\hat{\mathbf{y}}\right) \tag{3}$$

$$\ddot{\mathbf{p}}(t)=-\pi b^2\lambda_0\omega^2\left(-\sin\omega t\hat{\mathbf{x}}+\cos\omega t\hat{\mathbf{y}}\right) \tag{4}$$

$$\ddot{p}^2=\left(\pi b^2\lambda_0\omega^2\right)^2 \tag{5}$$

The total power radiated is therefore

$$P=\int\mathbf{S}\cdot d\mathbf{a}\cong\frac{\mu_0\ddot{p}^2}{6\pi c}=\frac{\mu_0\pi\lambda_0^2\omega^4b^4}{6c} \tag{6}$$

Calculating the fields and Poynting vector is more complicated, as they both change with time.