

POWER RADIATED BY A SPINNING RING OF CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 9.

Here's a generalization of the rotating dipole problem we did earlier. This time we have a circular ring with radius b with a linear charge distribution, at $t = 0$, of

$$\lambda = \lambda_0 \sin \phi \quad (1)$$

where ϕ is the azimuthal angle. The disk is set spinning with an angular velocity of ω .

Because $\sin(\phi + \pi) = -\sin \phi$, this disk is essentially a collection of dipoles with charges $\pm \lambda b d\phi$ separated by distance $2b$. Therefore, at time t , the dipole moment is

$$\mathbf{p}(t) = 2b^2 \lambda_0 \int_0^\pi \sin \phi [\cos(\omega t + \phi) \hat{\mathbf{x}} + \sin(\omega t + \phi) \hat{\mathbf{y}}] d\phi \quad (2)$$

$$= 2b^2 \lambda_0 \frac{\pi}{2} (-\sin \omega t \hat{\mathbf{x}} + \cos \omega t \hat{\mathbf{y}}) \quad (3)$$

$$\dot{\mathbf{p}}(t) = -\pi b^2 \lambda_0 \omega^2 (-\sin \omega t \hat{\mathbf{x}} + \cos \omega t \hat{\mathbf{y}}) \quad (4)$$

$$\ddot{p}^2 = (\pi b^2 \lambda_0 \omega^2)^2 \quad (5)$$

The total power radiated is therefore

$$P = \int \mathbf{S} \cdot d\mathbf{a} \cong \frac{\mu_0 \ddot{p}^2}{6\pi c} = \frac{\mu_0 \pi \lambda_0^2 \omega^4 b^4}{6c} \quad (6)$$

Calculating the fields and Poynting vector is more complicated, as they both change with time.