

## POWER RADIATED BY A SPINNING RING OF CHARGE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 9.

Here's a generalization of the rotating dipole problem we did earlier. This time we have a circular ring with radius  $b$  with a linear charge distribution, at  $t = 0$ , of

$$(0.1) \quad \lambda = \lambda_0 \sin \phi$$

where  $\phi$  is the azimuthal angle. The disk is set spinning with an angular velocity of  $\omega$ .

Because  $\sin(\phi + \pi) = -\sin \phi$ , this disk is essentially a collection of dipoles with charges  $\pm \lambda b d\phi$  separated by distance  $2b$ . Therefore, at time  $t$ , the dipole moment is

$$(0.2) \quad \mathbf{p}(t) = 2b^2 \lambda_0 \int_0^\pi \sin \phi [\cos(\omega t + \phi) \hat{\mathbf{x}} + \sin(\omega t + \phi) \hat{\mathbf{y}}] d\phi$$

$$(0.3) \quad = 2b^2 \lambda_0 \frac{\pi}{2} (-\sin \omega t \hat{\mathbf{x}} + \cos \omega t \hat{\mathbf{y}})$$

$$(0.4) \quad \ddot{\mathbf{p}}(t) = -\pi b^2 \lambda_0 \omega^2 (-\sin \omega t \hat{\mathbf{x}} + \cos \omega t \hat{\mathbf{y}})$$

$$(0.5) \quad \ddot{p}^2 = (\pi b^2 \lambda_0 \omega^2)^2$$

The total power radiated is therefore

$$(0.6) \quad P = \int \mathbf{S} \cdot d\mathbf{a} \cong \frac{\mu_0 \ddot{p}^2}{6\pi c} = \frac{\mu_0 \pi \lambda_0^2 \omega^4 b^4}{6c}$$

Calculating the fields and Poynting vector is more complicated, as they both change with time.