RADIATION FROM A CHARGE FALLING UNDER GRAVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 10.

If a charge falls under the influence of gravity, it accelerates and therefore radiates. This means that not all of the potential energy lost as the charge falls is converted to kinetic energy, so a charged object falls more slowly than an uncharged one. Will this difference be noticeable?

Suppose we drop a single electron from rest at $z=0$. After it has fallen to a position z, its dipole moment is

$$\mathbf{p}=ez\hat{\mathbf{z}} \tag{1}$$

(The dipole moment is in the $+z$ direction since the electron's charge is negative and it falls to a point $z<0$.) The power radiated is

$$P\cong\frac{\mu_0\ddot{p}^2}{6\pi c} \tag{2}$$

where

$$\begin{aligned}\ddot{p} &= e\ddot{z} &(3)\\ &= eg &(4)\end{aligned}$$

so

$$P=\frac{\mu_0 e^2 g^2}{6\pi c}=5.7\times10^{-52}\text{ J s}^{-1} \tag{5}$$

which is a constant.

To find how much energy is radiated as the electron falls, say, 1 cm, we need to know how long it takes the electron to fall 1 cm. If all its lost potential energy were converted to kinetic, then we get $v=gt$ and $d=\frac{1}{2}gt^2$. Since the power is very small, it's a safe bet that very little of the energy is radiated, so we can assume that $d=\frac{1}{2}gt^2$ and then check that our answer is consistent. From this we get

$$t=\sqrt{\frac{2\times 0.01}{9.8}}=0.045\text{ s} \tag{6}$$

so the total energy radiated is

$$Pt=2.46\times 10^{-53}\text{ J} \tag{7}$$

The potential energy lost is

$$V=mgh=0.01mg=8.92\times 10^{-32}\text{ J} \tag{8}$$

so the fraction of potential energy radiated is

$$\frac{Pt}{V}=2.76\times 10^{-22} \tag{9}$$

So hardly any of the energy is radiated.