RADIATION FROM A POINT CHARGE; THE LARMOR FORMULA

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 13.

We've seen that the fields produced by a moving point charge are

$$\mathbf{E}(\mathbf{r},t) = \frac{q\mathfrak{r}}{4\pi\varepsilon_0(\boldsymbol{\mathfrak{r}}\cdot\mathbf{u})^3}\left[\left(c^2-v^2\right)\mathbf{u}+\boldsymbol{\mathfrak{r}}\times(\mathbf{u}\times\mathbf{a})\right] \tag{1}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\boldsymbol{\mathfrak{r}}}\times\mathbf{E}(\mathbf{r},t) \tag{2}$$

where $\boldsymbol{\mathfrak{r}}$ is the vector from the charge to the observation point $\mathbf{r}$ and $\mathbf{u} = c\hat{r}-\mathbf{v}$.

If we're interested only in the power radiated by a moving point charge, we need keep only those terms in the fields that depend on $\frac{1}{r}$ and discard higher order terms such as $\frac{1}{r^2}$. This is because the power radiated depends on the product of the fields via the Poynting vector, and any terms in the Poynting vector of order $\frac{1}{r^3}$ or higher will go to zero when integrated over a large sphere. It's actually more convenient to use a coordinate system centred on the moving charge, so that $\mathbf{r}$ and $\boldsymbol{\mathfrak{r}}$ are actually the same, and $\mathfrak{r}$ becomes the radius of the enclosing sphere. Looking at 1, we see that there is a factor of order $\frac{1}{\mathfrak{r}^2}$ out front, and of the two terms inside the square brackets, only the second term contains another factor of $\mathfrak{r}$. Combining these two terms means that the only term that will contribute to radiation from the point charge is the second one, so we have

$$\mathbf{E}_{rad} = \frac{q\mathfrak{r}}{4\pi\varepsilon_0(\boldsymbol{\mathfrak{r}}\cdot\mathbf{u})^3}\boldsymbol{\mathfrak{r}}\times(\mathbf{u}\times\mathbf{a}) \tag{3}$$

As it depends on the acceleration and results in radiation, this term is known as either the *acceleration field* or *radiation field.* The first term in 1 also results in energy flux generated by the moving charge, but as it goes as $1/\mathfrak{r}^2$ it is a localized flux, and does not contribute to radiation as it drops to zero for large r.

From here, the derivation of the formula for the total power radiated is fairly straightforward and is given in detail by Griffiths in his section 11.2.1.

The only further assumption that is made is that $\mathbf{v}=0$ at the particular instant of time that we're considering. [Recall that it is possible for the velocity of a particle to be zero while its acceleration is non-zero, as with a mass oscillating on a spring when it reaches the high and low points of its trajectory.] This assumption gives the *Larmor formula* for the power radiated by a point charge:

$$P=\frac{\mu_0 q^2 a^2}{6\pi c} \tag{4}$$

Note that this formula has an implicit time dependence through the acceleration $a=a\left(t_r\right)=a\left(t-\frac{\mathfrak{r}}{c}\right)$, although the other parameters are all constants. Since we're dealing with a point charge, there is only one retarded time that we need to keep track of. The radiation detected at the enclosing sphere, which is centred on the charge (so it moves as the charge moves) is the radiation that left the charge at time $t-\frac{\mathfrak{r}}{c}$.

Although the Larmor formula was derived by assuming that $v=0$ it is actually a good approximation for all non-relativistic speeds.

Example. As an example, suppose we have an electron moving at a thermal speed of $v_0=10^5\text{ m s}^{-1}$ (so $v\ll c$) within a solid, such as a metal, and by colliding with an atom it experiences a constant deceleration so that it comes to rest after travelling $d=3\times10^{-9}$ m. To find the power radiated by the electron using the Larmor formula, we need to know how long the deceleration takes. Since a is constant, we have

$$t=\frac{v_0}{a} \tag{5}$$

The total energy radiated over this time is

$$\begin{aligned} E &= Pt && (6)\\ &= \frac{\mu_0 q^2 a^2}{6\pi c}\frac{v_0}{a} && (7)\\ &= \frac{\mu_0 q^2 a v_0}{6\pi c} && (8) \end{aligned}$$

The fraction of the electron's initial kinetic energy radiated away is

$$\begin{aligned} f &= \frac{2E}{mv_0^2} && (9)\\ &= \frac{\mu_0 q^2 a}{3\pi m c v_0} && (10) \end{aligned}$$

To find a we use the formula

$$d = \frac{1}{2}at^2 = \frac{v_0^2}{2a} \tag{11}$$

$$a = \frac{v_0^2}{2d} \tag{12}$$

$$= 1.67\times10^{18}\text{ m s}^{-2} \tag{13}$$

[Yes, that's an enormous deceleration!]
Plugging in the other constants in 10 we get

$$f = 2\times10^{-10} \tag{14}$$

Thus the amount of energy lost to radiation due to electronic collisions is very small.

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