

## RADIATION REACTION: A FEW EXAMPLES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 17.

Here are a few simple examples of the radiation reaction force, calculated using the Abraham-Lorentz formula.

**Example 1.** A charge  $q$  moves in a circle of radius  $R$  with constant speed  $v$ . The reaction force is given by

$$\mathbf{F}_{rad} = \frac{\mu_0 q^2}{6\pi c} \dot{\mathbf{a}} \quad (1)$$

For uniform circular motion, the acceleration is always directed towards the centre of the circle, so

$$\mathbf{a} = -\frac{v^2}{R} \hat{\mathbf{r}} \quad (2)$$

$$= -\frac{v^2}{R^2} \mathbf{r} \quad (3)$$

$$\dot{\mathbf{a}} = -\frac{v^2}{R^2} \mathbf{v} \quad (4)$$

$$\mathbf{F}_{rad} = -\frac{\mu_0 q^2 v^2}{6\pi c R^2} \mathbf{v} \quad (5)$$

The force we need to apply to counter the reaction force is just the negative of this, so

$$\mathbf{F}_e = \frac{\mu_0 q^2 v^2}{6\pi c R^2} \mathbf{v} \quad (6)$$

The power generated by applying this force is

$$P_e = \mathbf{F}_e \cdot \mathbf{v} = \frac{\mu_0 q^2 v^4}{6\pi c R^2} \quad (7)$$

The Larmor formula for radiated power is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad (8)$$

$$= \frac{\mu_0 q^2 v^4}{6\pi c R^2} \quad (9)$$

$$= P_e \quad (10)$$

So the power we must exert to counter the reaction force is equal to the power radiated away.

**Example 2.** Now suppose we have a charge on a spring which moves with simple harmonic motion according to

$$\mathbf{r}(t) = A\hat{\mathbf{z}}\cos\omega t \quad (11)$$

$$\mathbf{v}(t) = -A\omega\hat{\mathbf{z}}\sin\omega t \quad (12)$$

$$\mathbf{a}(t) = -A\omega^2\hat{\mathbf{z}}\cos\omega t \quad (13)$$

$$\dot{\mathbf{a}}(t) = A\omega^3\hat{\mathbf{z}}\sin\omega t \quad (14)$$

The reaction force is now

$$\mathbf{F}_{rad} = \frac{\mu_0 q^2 A \omega^3}{6\pi c} \hat{\mathbf{z}} \sin\omega t \quad (15)$$

and the force  $\mathbf{F}_e$  we need to apply to counter it is

$$\mathbf{F}_e = -\frac{\mu_0 q^2 A \omega^3}{6\pi c} \hat{\mathbf{z}} \sin\omega t \quad (16)$$

giving a power of

$$P_e = \mathbf{F}_e \cdot \mathbf{v} \quad (17)$$

$$= \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \sin^2\omega t \quad (18)$$

The radiated power is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad (19)$$

$$= \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \cos^2\omega t \quad (20)$$

Thus in this case, the applied power is not equal to the radiated power at each instant of time, but remember that the Abraham-Lorentz formula was

derived by taking the average power over a period of time after which the system returns to its initial state. If we average these two powers over one cycle, we get

$$\langle P_e \rangle = \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin^2 \omega t \, dt \quad (21)$$

$$= \frac{\mu_0 q^2 A^2 \omega^4}{12\pi c} \quad (22)$$

$$= \langle P \rangle \quad (23)$$

Thus, on average, the powers are equal.

**Example 3.** For a charge falling in a constant gravitational field with acceleration  $g$ ,  $\hat{\mathbf{a}} = 0$  so the reaction force is zero. However, since the acceleration is not zero, the charge does radiate with a power of

$$P = \frac{\mu_0 q^2 g^2}{6\pi c} \quad (24)$$

In this case, there is no time interval after which the charge returns to its initial state, so we can't average the power over any time interval and the Abraham-Lorentz formula isn't valid.