

RADIATION REACTION: A FEW EXAMPLES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 17.

Here are a few simple examples of the radiation reaction force, calculated using the Abraham-Lorentz formula.

Example 1. A charge q moves in a circle of radius R with constant speed v . The reaction force is given by

$$(0.1) \quad \mathbf{F}_{rad} = \frac{\mu_0 q^2}{6\pi c} \dot{\mathbf{a}}$$

For uniform circular motion, the acceleration is always directed towards the centre of the circle, so

$$(0.2) \quad \mathbf{a} = -\frac{v^2}{R} \hat{\mathbf{r}}$$

$$(0.3) \quad = -\frac{v^2}{R^2} \mathbf{r}$$

$$(0.4) \quad \dot{\mathbf{a}} = -\frac{v^2}{R^2} \dot{\mathbf{v}}$$

$$(0.5) \quad \mathbf{F}_{rad} = -\frac{\mu_0 q^2 v^2}{6\pi c R^2} \mathbf{v}$$

The force we need to apply to counter the reaction force is just the negative of this, so

$$(0.6) \quad \mathbf{F}_e = \frac{\mu_0 q^2 v^2}{6\pi c R^2} \mathbf{v}$$

The power generated by applying this force is

$$(0.7) \quad P_e = \mathbf{F}_e \cdot \mathbf{v} = \frac{\mu_0 q^2 v^4}{6\pi c R^2}$$

The Larmor formula for radiated power is

$$(0.8) \quad P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$(0.9) \quad = \frac{\mu_0 q^2 v^4}{6\pi c R^2}$$

$$(0.10) \quad = P_e$$

So the power we must exert to counter the reaction force is equal to the power radiated away.

Example 2. Now suppose we have a charge on a spring which moves with simple harmonic motion according to

$$(0.11) \quad \mathbf{r}(t) = A\hat{\mathbf{z}}\cos\omega t$$

$$(0.12) \quad \mathbf{v}(t) = -A\omega\hat{\mathbf{z}}\sin\omega t$$

$$(0.13) \quad \mathbf{a}(t) = -A\omega^2\hat{\mathbf{z}}\cos\omega t$$

$$(0.14) \quad \dot{\mathbf{a}}(t) = A\omega^3\hat{\mathbf{z}}\sin\omega t$$

The reaction force is now

$$(0.15) \quad \mathbf{F}_{rad} = \frac{\mu_0 q^2 A \omega^3}{6\pi c} \hat{\mathbf{z}} \sin\omega t$$

and the force \mathbf{F}_e we need to apply to counter it is

$$(0.16) \quad \mathbf{F}_e = -\frac{\mu_0 q^2 A \omega^3}{6\pi c} \hat{\mathbf{z}} \sin\omega t$$

giving a power of

$$(0.17) \quad P_e = \mathbf{F}_e \cdot \mathbf{v}$$

$$(0.18) \quad = \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \sin^2\omega t$$

The radiated power is

$$(0.19) \quad P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$(0.20) \quad = \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \cos^2\omega t$$

Thus in this case, the applied power is not equal to the radiated power at each instant of time, but remember that the Abraham-Lorentz formula was derived by taking the average power over a period of time after which the system returns to its initial state. If we average these two powers over one cycle, we get

$$(0.21) \quad \langle P_e \rangle = \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin^2 \omega t$$

$$(0.22) \quad = \frac{\mu_0 q^2 A^2 \omega^4}{12\pi c}$$

$$(0.23) \quad = \langle P \rangle$$

Thus, on average, the powers are equal.

Example 3. For a charge falling in a constant gravitational field with acceleration g , $\dot{\mathbf{a}} = 0$ so the reaction force is zero. However, since the acceleration is not zero, the charge does radiate with a power of

$$(0.24) \quad P = \frac{\mu_0 q^2 g^2}{6\pi c}$$

In this case, there is no time interval after which the charge returns to its initial state, so we can't average the power over any time interval and the Abraham-Lorentz formula isn't valid.