ABRAHAM-LORENTZ FORMULA - PHYSICAL BASIS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 20.

We've seen that the radiation reaction force is given by the Abraham-Lorentz formula

$$\mathbf{F}_{rad}=\frac{\mu_0 q^2}{6\pi c}\dot{\mathbf{a}} \tag{1}$$

This formula was derived for the special case of a particle that returns to its initial state after some time, and is calculated by taking the average work done over that time period.

The formula is derived from the conservation of energy, but it doesn't really tell us much about how the force originates. As with most things about radiation reaction, this origin isn't particularly well understood, but it appears that the force is a form of 'self-force', that is, a force felt by a charge in motion due to the field it emitted at a previous time. If you think of a charge moving along some trajectory, then after it has moved some distance Δx, it will feel the effect of the field that it emitted at a previous time of $\Delta x/c$, since the fields travel at the speed of light.

In his section 11.2.3, Griffiths derives the Abraham-Lorentz formula by considering a charge q moving along the x axis. In order to deduce the force of a charge on itself, he splits the charge into a dumbbell whose axis is perpendicular to the direction of motion. There is a charge of $\frac{q}{2}$ at each end of the dumbbell, and these two half-charges are separated by a distance d, which is imagined to tend to zero as we consider the limit of a point charge. By considering the force of the bottom charge on the top charge, he shows that this force comes out to

$$F=\frac{\mu_0 q^2 \dot{a}}{24\pi c} \tag{2}$$

(This formula applies in the special case where the charge is instantaeously at rest at the retarded time when the field felt at the current time was emitted.) By symmetry, there is an equal force exerted by the top charge on the bottom one, so the total force due to these two interactions is double the force above, so

$$F_{rad}^{int} = \frac{\mu_0 q^2 \dot{a}}{12\pi c} \tag{3}$$

However, there is also a force felt by each charge from *its own* field that it emitted at the retarded time. Applying the formula 1, the self-force felt by the upper charge is

$$F_{self_1} = \frac{\mu_0 (q/2)^2 \dot{a}}{6\pi c} = \frac{\mu_0 q^2 \dot{a}}{24\pi c} \tag{4}$$

Again, from symmetry, there is an equal self-force contribution from the lower charge, so

$$\begin{aligned}
F_{self_2} &= \frac{\mu_0 (q/2)^2 \dot{a}}{6\pi c} = \frac{\mu_0 q^2 \dot{a}}{24\pi c} && (5)\\
F_{self} &= F_{self_1} + F_{self_2} && (6)\\
&= \frac{\mu_0 q^2 \dot{a}}{12\pi c} && (7)\\
F_{total} &= F_{rad}^{int} + F_{self} && (8)\\
&= \frac{\mu_0 q^2 \dot{a}}{6\pi c} && (9)
\end{aligned}$$

This derivation isn't very convincing, since we used the Abraham-Lorentz formula to prove itself, so our reasing is circular. Another approach is to apply the original cross-charge force to a strip of charge smeared out over the entire length L of the dumbbell. That is, for each segment dy on the dumbbell, we consider the force felt by fields emitted by charge segments from the entire strip at the retarded time. If the linear charge density is λ then

$$\int_0^L \lambda \, dy = \lambda L = q \tag{10}$$

If we now consider a pair of segments on this strip, then the charge on each segment is $\lambda \, dy$, which replaces $q/2$ in 3, so the reaction force on these two segments is

$$dF = \frac{\mu_0 (q/2)^2 \dot{a}}{3\pi c} = \frac{\mu_0 \lambda^2 \dot{a}}{3\pi c} dy_1 dy_2 \tag{11}$$

The total reaction force on the strip is

$$F = \frac{1}{2}\frac{\mu_0\lambda^2\dot{a}}{3\pi c}\int_0^L\int_0^L dy_1 dy_2 \tag{12}$$

$$= \frac{\mu_0(L\lambda)^2\dot{a}}{6\pi c} \tag{13}$$

$$= \frac{\mu_0 q^2\dot{a}}{6\pi c} \tag{14}$$

where we introduced the factor of $\frac{1}{2}$ in the first line to avoid double-counting each pair of segments.

Although this 'derivation' does give the Abraham-Lorentz formula, it's not a general proof, because if we use different configurations of charge (such as a strip of charge whose axis is parallel to the direction of motion, or a sphere of charge) we don't get the Abraham-Lorentz formula out of the derivation. This is yet another problem with this formula to add to preacceleration and runaway acceleration.

PINGBACKS

Pingback: Physical basis of the radiation reaction force

Pingback: Self-force on a dipole in hyperbolic motion