RADIATION FROM A RADIO STATION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 22.

A radio station's transmitter consists of a circular current loop acting as a magnetic dipole. The dipole is on a radio tower 200 m above the ground, and is emitting a total power of 3.5×10^{4} watts. We can find the position on the ground receiving the maximum power, although unlike the previous problem, we're concerned only with the actual magnitude of the power and not with the power per unit area of ground. The formula for intensity is

$$\langle\mathbf{S}\rangle=\frac{\mu_{0}m_{0}^{2}\omega^{4}\sin^{2}\theta}{32\pi^{2}c^{3}r^{2}}\hat{\mathbf{r}} \tag{0.1}$$

For a location on the ground that is a distance R from the base of the tower,

$$\sin\theta = \frac{R}{\sqrt{R^{2}+h^{2}}} \tag{0.2}$$

$$r^{2} = R^{2}+h^{2} \tag{0.3}$$

so the intensity is

$$\langle\mathbf{S}\rangle=\frac{\mu_{0}m_{0}^{2}\omega^{4}}{32\pi^{2}c^{3}}\frac{R^{2}}{\left(R^{2}+h^{2}\right)^{2}}\hat{\mathbf{r}} \tag{0.4}$$

The position on the ground receiving maximum intensity is determined from

$$\frac{d\langle S\rangle}{dR} = 0 \tag{0.5}$$

$$= -\frac{4R^{3}}{\left(R^{2}+h^{2}\right)^{3}}+\frac{2R}{\left(R^{2}+h^{2}\right)^{2}} \tag{0.6}$$

$$R_{max} = h \tag{0.7}$$

and the intensity at this location is

$$\langle S\rangle_{max} = \frac{\mu_0 m_0^2 \omega^4}{128\pi^2 c^3 h^2} \tag{0.8}$$

The total power is obtained by integrating over a sphere of radius r:

$$\langle P\rangle = \int \mathbf{S}\cdot d\mathbf{a} \tag{0.9}$$

$$= \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \int_0^{\pi} \int_0^{2\pi} \frac{\sin^2\theta}{r^2} r^2 \sin\theta d\phi d\theta \tag{0.10}$$

$$= \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \tag{0.11}$$

Therefore the maximum intensity can be written in terms of the power:

$$\langle S\rangle_{max} = \frac{12}{128\pi}\frac{\langle P\rangle}{h^2} \tag{0.12}$$

$$= \frac{12}{128\pi}\frac{3.5\times 10^4}{200^2} \tag{0.13}$$

$$= 2.6\times 10^{-2} \text{ watts m}^{-2} \tag{0.14}$$

$$= 2.6\times 10^{-6} \text{ watts cm}^{-2} \tag{0.15}$$