RADIATION FROM THE MAGNETIC DIPOLES OF EARTH AND PULSARS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 23.

Because Earth's magnetic north pole is not at the same place as the orbital north pole, a component of Earth's magnetic dipole rotates as the planet rotates each day. This gives rise to radiation being emitted. Should we worry that Earth is losing a lot of energy in this way?

Suppose the angle (latitude difference) between true north and magnetic north is ψ and the magnetic dipole moment is $\mathbf{M}$. Then the component of $\mathbf{M}$ that rotates has magnitude $M\sin\psi$ so we can write this component as

$$\mathbf{M}_{rot}=M\sin\psi\left[\hat{\mathbf{x}}\cos\omega t+\hat{\mathbf{y}}\sin\omega t\right] \tag{0.1}$$

The power radiated by a time-varying magnetic dipole is

$$P=\frac{\mu_0\ddot{m}^2}{6\pi c^3} \tag{0.2}$$

where $m(t)$ is the magnitude of the magnetic dipole moment. In our case

$$\ddot{m}=\omega^2M\sin\psi\left(\cos^2\omega t+\sin^2\omega t\right)^{1/2}=\omega^2M\sin\psi \tag{0.3}$$

so the power radiated is

$$P=\frac{\mu_0\omega^4M^2\sin^2\psi}{6\pi c^3} \tag{0.4}$$

The magnetic field due to a dipole moment $\mathbf{M}$ is

$$\mathbf{B}=\frac{\mu_0}{4\pi r^3}\left[3\left(\mathbf{M}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}}-\mathbf{M}\right] \tag{0.5}$$

Taking Earth's field strength at the equator to be around 0.5 Gauss $=5\times10^{-5}$ Tesla and taking $\mathbf{M}\cdot\hat{\mathbf{r}}=0$ at the equator and the radius of Earth to be $r=6.37\times10^6$ m, we get an estimate of Earth's magnetic dipole moment:

$$M \approx \frac{4\pi r^3 B}{\mu_0} = 1.3 \times 10^{23} \text{Amp m}^2 \tag{0.6}$$

Using $\psi = 11^\circ$ and $\omega = 2\pi/(60 \times 60 \times 24) = 7.27 \times 10^{-5}\ \text{s}^{-1}$ we get

$$P = 4 \times 10^{-5} \text{ watts} \tag{0.7}$$

We needn't worry about Earth losing a significant amount of energy through radiation from its magnetic field.

For a pulsar, however, it's a different story. A typical pulsar has a radius of around 10 km $= 10^4$ m, a magnetic field strength of 10^8 Tesla and a rotation period of 10^{-3} s giving $\omega = 2\pi \times 10^3\ \text{s}^{-1}$. This gives

$$M = 10^{27} \text{ Amp m}^2 \tag{0.8}$$

$$P = 3.8 \times 10^{36} \sin^2 \psi \text{ watts} \tag{0.9}$$

Taking an average value of $\frac{1}{2}$ for $\sin^2 \psi$ we get a power of around 2×10^{36} watts. By comparison, the Sun produces 'only' around 4×10^{26} wattsso a pulsar produces the output of 5 billion suns from its magnetic dipole radiation alone.

Not all pulsars generate this much power, though. The period of the crab nebula pulsar is around 33 ms, so (assuming the radius and magnetic field are the same as above) this gives a power of around 2×10^{30} watts which is still pretty impressive.