RADIATION FROM A POINT CHARGE NEAR A CONDUCTING PLANE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 25.

For another example of radiation, we can return to the example of the point charge q at a distance z from an infinite conducting plane, which we looked at in electrostatics as an example of the method of images. We saw then that we could replace the conducting plane by an equal and opposite charge $-q$ at position $-z$. This makes the xy plane (the plane of the conductor) an equipotential surface with $V = 0$, so it satisfies the boundary conditions for the half-space $z \geq 0$.

Because the potentials of the original and image systems are the same in the physical region (the region outside the conductor), so too are the fields, and since the force on the charge is determined by the fields, the forces in the two systems are also equal, so we have

$$\mathbf{F} = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{(2z)^2}\hat{\mathbf{z}} \tag{1}$$

where the negative sign indicates that the force is towards the plane (or towards the image charge, in the image system).

If the charge is not restrained, this force will cause it to accelerate, so

$$\mathbf{a} = \frac{\mathbf{F}}{m} = -\frac{1}{4\pi\epsilon_0}\frac{q^2}{(2z)^2 m}\hat{\mathbf{z}} \tag{2}$$

This acceleration will cause the charge to radiate, and we can calculate the dipole radiation from the formula

$$P \cong \frac{\mu_0 \ddot{p}^2}{6\pi c} \tag{3}$$

where p is the dipole moment of the charge distribution. The dipole moment of the charge and its image is

$$\mathbf{p} = 2qz\hat{\mathbf{z}} \tag{4}$$

since the two charges are separated by a distance $2z$. Therefore

$$\ddot{\mathbf{p}} = 2q\ddot{z}\hat{\mathbf{z}} = 2q\mathbf{a} = -\frac{1}{2\pi\epsilon_0}\frac{q^3}{(2z)^2 m}\hat{\mathbf{z}} \tag{5}$$

so the power is

$$\begin{aligned} P &\cong \frac{\mu_0}{6\pi c}\left(\frac{1}{2\pi\epsilon_0}\frac{q^3}{(2z)^2 m}\right)^2 &(6)\\ &= \frac{\mu_0}{6\pi c}\left(\frac{c^2\mu_0}{2\pi}\frac{q^3}{(2z)^2 m}\right)^2 &(7)\\ &= \left(\frac{\mu_0 c q^2}{4\pi}\right)^3 \frac{1}{6m^2 z^4} &(8)\end{aligned}$$

Although this is the answer given in Griffiths's book (in his problem 11.25), I'm not convinced it's actually correct. The *actual* charge distribution that we calculated when studying the method of images, that is, using the point charge q at position z and the surface charge density, given by

$$\sigma = \frac{-qz}{2\pi\left(r^2+z^2\right)^{3/2}} \tag{9}$$

where r is the radial distance on the xy plane measured from the point directly beneath the point charge. If we take the origin to be the point in the xy plane directly beneath the point charge, the dipole moment of the surface charge comes out to zero (by symmetry; for each charge element at a point $\mathbf{r}$ there is an equal charge element at point $-\mathbf{r}$ so their dipole moments cancel out), while the dipole moment of the point charge is $qz\hat{\mathbf{z}}$, so the total dipole moment of the point charge + surface charge is also $qz\hat{\mathbf{z}}$. Since the dipole moment is lower by a factor of 2, the total power radiated is lower by a factor of 4, and would seem to be

$$P = \left(\frac{\mu_0 c q^2}{4\pi}\right)^3 \frac{1}{24m^2 z^4} \tag{10}$$

If we assume that the half space $z \le 0$ is entirely occupied by the conductor, there can be no free charge anywhere except on the surface of the conductor (and of course the point charge itself). I suspect the problem may have something to do with the fact that the method of images really applies only in electrostatics, though I'm not entirely sure. Comments welcome.