RADIATION REACTION WITH A DELTA-FUNCTION EXTERNAL FORCE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 28.

Here's another example of applying an external force to a charge feeling the radiation reaction force. In general, a charge's acceleration obeys the differential equation

$$a=\tau\dot{a}+\frac{F}{m} \tag{1}$$

where F is the external force and

$$\tau\equiv\frac{\mu_0 q^2}{6\pi mc} \tag{2}$$

Suppose now that the force is a delta function:

$$F=k\delta(t) \tag{3}$$

for some constant k. In the earlier post, we showed that if F is finite everywhere, then a must be continuous everywhere. However, here F is not finite at $t=0$. As before we start by integrating 1 over a small time interval around $t=0$:

$$\int_{-\varepsilon}^{\varepsilon}a\,dt=\tau\left[a(\varepsilon)-a(-\varepsilon)\right]+\frac{1}{m}\int_{-\varepsilon}^{\varepsilon}F\,dt \tag{4}$$

Provided that a is finite everywhere, the integral on the LHS goes to zero as $\varepsilon\to 0$ so we're left with

$$\tau\Delta a = -\frac{1}{m}\int_{-\varepsilon}^{\varepsilon} F\,dt \tag{5}$$

$$= -\frac{k}{m}\int_{-\varepsilon}^{\varepsilon} \delta(t)\,dt \tag{6}$$

$$= -\frac{k}{m} \tag{7}$$

$$\Delta a = -\frac{k}{m\tau} \tag{8}$$

We can repeat the calculations we did earlier to check that energy is conserved here. Since $F = 0$ everywhere except $t = 0$, the general solution of 1 is

$$a(t) = \begin{cases} a_0 e^{t/\tau} & t < 0 \\ a_1 e^{t/\tau} & t > 0 \end{cases} \tag{9}$$

If we eliminate the runaway acceleration for $t > 0$ by requiring $a_1 = 0$ then the condition 8 requires $a_0 = k/m\tau$, so

$$a(t) = \begin{cases} \frac{k}{m\tau} e^{t/\tau} & t < 0 \\ 0 & t > 0 \end{cases} \tag{10}$$

By requiring $v = 0$ at $t = -\infty$ and that v is continuous at $t = 0$ we get

$$v(t) = \begin{cases} \frac{k}{m} e^{t/\tau} & t < 0 \\ \frac{k}{m} & t > 0 \end{cases} \tag{11}$$

The work done by the force is

$$W = \int_{-\infty}^{\infty} F v\,dt \tag{12}$$

$$= k\int_{-\infty}^{\infty} \delta(t)\,v\,dt \tag{13}$$

$$= kv(0) \tag{14}$$

$$= \frac{k^2}{m} \tag{15}$$

The energy radiated R is given by integrating the Larmor formula

$$P=\frac{\mu_0 q^2 a^2}{6\pi c}=m\tau a^2 \tag{16}$$

so we get

$$R = m\tau\int_{-\infty}^{0}\left(\frac{k}{m\tau}\right)^2 e^{2t/\tau}dt \tag{17}$$

$$= \frac{k^2}{2m} \tag{18}$$

The final kinetic energy is

$$K=\frac{1}{2}m\frac{k^2}{m^2}=\frac{k^2}{2m} \tag{19}$$

Thus

$$W=R+K \tag{20}$$

and energy is conserved.

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