RADIATION REACTION WITH A DELTA-FUNCTION EXTERNAL FORCE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 28.

Here's another example of applying an external force to a charge feeling the radiation reaction force. In general, a charge's acceleration obeys the differential equation

$$a=\tau\dot{a}+\frac{F}{m} \tag{1}$$

where F is the external force and

$$\tau\equiv\frac{\mu_0 q^2}{6\pi mc} \tag{2}$$

Suppose now that the force is a delta function:

$$F=k\delta\left(t\right) \tag{3}$$

for some constant k. In the earlier post, we showed that if F is finite everywhere, then a must be continuous everywhere. However, here F is not finite at $t=0$. As before we start by integrating 1 over a small time interval around $t=0$:

$$\int_{-\varepsilon}^{\varepsilon}a\,dt=\tau\left[a\left(\varepsilon\right)-a\left(-\varepsilon\right)\right]+\frac{1}{m}\int_{-\varepsilon}^{\varepsilon}F\,dt \tag{4}$$

Provided that a is finite everywhere, the integral on the LHS goes to zero as $\varepsilon\to0$ so we're left with

$$\begin{aligned}
\tau\Delta a &= -\frac{1}{m}\int_{-\varepsilon}^{\varepsilon} F\,dt & (5)\\
&= -\frac{k}{m}\int_{-\varepsilon}^{\varepsilon} \delta(t)\,dt & (6)\\
&= -\frac{k}{m} & (7)\\
\Delta a &= -\frac{k}{m\tau} & (8)
\end{aligned}$$

We can repeat the calculations we did earlier to check that energy is conserved here. Since $F=0$ everywhere except $t=0$, the general solution of 1 is

$$a(t)=\begin{cases} a_0 e^{t/\tau} & t<0\\ a_1 e^{t/\tau} & t>0\end{cases} \tag{9}$$

If we eliminate the runaway acceleration for $t>0$ by requiring $a_1=0$ then the condition 8 requires $a_0=k/m\tau$, so

$$a(t)=\begin{cases} \frac{k}{m\tau} e^{t/\tau} & t<0\\ 0 & t>0\end{cases} \tag{10}$$

By requiring $v=0$ at $t=-\infty$ and that v is continuous at $t=0$ we get

$$v(t)=\begin{cases} \frac{k}{m} e^{t/\tau} & t<0\\ \frac{k}{m} & t>0\end{cases} \tag{11}$$

The work done by the force is

$$\begin{aligned}
W &= \int_{-\infty}^{\infty} Fv\,dt & (12)\\
&= k\int_{-\infty}^{\infty} \delta(t)\,v\,dt & (13)\\
&= kv(0) & (14)\\
&= \frac{k^2}{m} & (15)
\end{aligned}$$

The energy radiated R is given by integrating the Larmor formula

$$P=\frac{\mu_0 q^2 a^2}{6\pi c}=m\tau a^2 \tag{16}$$

so we get

$$\begin{aligned} R &= m\tau\int_{-\infty}^{0}\left(\frac{k}{m\tau}\right)^{2}e^{2t/\tau}dt && (17)\\ &= \frac{k^{2}}{2m} && (18)\end{aligned}$$

The final kinetic energy is

$$K=\frac{1}{2}m\frac{k^{2}}{m^{2}}=\frac{k^{2}}{2m} \tag{19}$$

Thus

$$W=R+K \tag{20}$$

and energy is conserved.

PINGBACKS

Pingback: Tunnelling through a potential barrier with the radiation reaction force