RADIATION FROM A CHARGE IN HYPERBOLIC MOTION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 31.

A particle moving along the x axis in hyperbolic motion has a position given by

$$x(t)=\sqrt{b^2+c^2t^2} \tag{1}$$

where b is a constant giving the closest approach to the origin. If the particle has charge q then its radiation is given by

$$P=\frac{\mu_0 q^2\gamma^6 a^2}{6\pi c} \tag{2}$$

To calculate this we need

$$\begin{aligned}
v(t) &= \frac{tc^2}{\sqrt{b^2+c^2t^2}}=\frac{tc^2}{x} && (3)\\
a(t) &= \frac{c^2}{x}-\frac{c^4t^2}{x^3}=\frac{c^2b^2}{x^3} && (4)\\
\gamma &= \frac{1}{\sqrt{1-v^2/c^2}} && (5)\\
&= \frac{1}{\sqrt{1-\frac{t^2c^2}{x^2}}} && (6)\\
&= \frac{x}{b} && (7)
\end{aligned}$$

The power is therefore

$$\begin{aligned}
P &= \frac{\mu_0 q^2}{6\pi c}\left(\frac{x}{b}\right)^6\left(\frac{c^2b^2}{x^3}\right)^2 && (8)\\
&= \frac{\mu_0 q^2 c^3}{6\pi b^2} && (9)
\end{aligned}$$

Thus the radiated power is constant as the charge moves along its trajectory.

The radiation reaction force is given by

$$F_{rad} = \frac{\mu_0 q^2 \gamma^4}{6\pi c}\left(\dot{a} + \frac{3a^2\gamma^2 v}{c^2}\right) \tag{10}$$

for which we need

$$\dot{a} = -\frac{3c^4 b^2 t}{x^5} \tag{11}$$

We get

$$\frac{3a^2\gamma^2 v}{c^2} = \frac{3}{c^2}\left(\frac{c^2 b^2}{x^3}\right)^2 \left(\frac{x}{b}\right)^2 \left(\frac{tc^2}{x}\right) \tag{12}$$

$$= \frac{3c^4 b^2 t}{x^5} \tag{13}$$

$$= -\dot{a} \tag{14}$$

$$F_{rad} = 0 \tag{15}$$

This is a rather curious case in that although the charge radiates, there is no radiation reaction force.