

## RADIATION FROM A CHARGE IN HYPERBOLIC MOTION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 31.

A particle moving along the  $x$  axis in hyperbolic motion has a position given by

$$x(t) = \sqrt{b^2 + c^2 t^2} \quad (1)$$

where  $b$  is a constant giving the closest approach to the origin. If the particle has charge  $q$  then its radiation is given by

$$P = \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c} \quad (2)$$

To calculate this we need

$$v(t) = \frac{tc^2}{\sqrt{b^2 + c^2 t^2}} = \frac{tc^2}{x} \quad (3)$$

$$a(t) = \frac{c^2}{x} - \frac{c^4 t^2}{x^3} = \frac{c^2 b^2}{x^3} \quad (4)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (5)$$

$$= \frac{1}{\sqrt{1 - \frac{t^2 c^2}{x^2}}} \quad (6)$$

$$= \frac{x}{b} \quad (7)$$

The power is therefore

$$P = \frac{\mu_0 q^2}{6\pi c} \left(\frac{x}{b}\right)^6 \left(\frac{c^2 b^2}{x^3}\right)^2 \quad (8)$$

$$= \frac{\mu_0 q^2 c^3}{6\pi b^2} \quad (9)$$

Thus the radiated power is constant as the charge moves along its trajectory.

The radiation reaction force is given by

$$F_{rad} = \frac{\mu_0 q^2 \gamma^4}{6\pi c} \left( \dot{a} + \frac{3a^2 \gamma^2 v}{c^2} \right) \quad (10)$$

for which we need

$$\dot{a} = -\frac{3c^4 b^2 t}{x^5} \quad (11)$$

We get

$$\frac{3a^2 \gamma^2 v}{c^2} = \frac{3}{c^2} \left( \frac{c^2 b^2}{x^3} \right)^2 \left( \frac{x}{b} \right)^2 \left( \frac{tc^2}{x} \right) \quad (12)$$

$$= \frac{3c^4 b^2 t}{x^5} \quad (13)$$

$$= -\dot{a} \quad (14)$$

$$F_{rad} = 0 \quad (15)$$

This is a rather curious case in that although the charge radiates, there is no radiation reaction force.