RADIATION FROM A CHARGE IN HYPERBOLIC MOTION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 11, Post 31.

A particle moving along the x axis in hyperbolic motion has a position given by

$$x(t)=\sqrt{b^{2}+c^{2}t^{2}} \tag{0.1}$$

where b is a constant giving the closest approach to the origin. If the particle has charge q then its radiation is given by

$$P=\frac{\mu_{0}q^{2}\gamma^{6}a^{2}}{6\pi c} \tag{0.2}$$

To calculate this we need

$$\begin{aligned}
v(t) &= \frac{tc^{2}}{\sqrt{b^{2}+c^{2}t^{2}}}=\frac{tc^{2}}{x} && (0.3)\\
a(t) &= \frac{c^{2}}{x}-\frac{c^{4}t^{2}}{x^{3}}=\frac{c^{2}b^{2}}{x^{3}} && (0.4)\\
\gamma &= \frac{1}{\sqrt{1-v^{2}/c^{2}}} && (0.5)\\
&= \frac{1}{\sqrt{1-\frac{t^{2}c^{2}}{x^{2}}}} && (0.6)\\
&= \frac{x}{b} && (0.7)
\end{aligned}$$

The power is therefore

$$\begin{aligned}
P &= \frac{\mu_{0}q^{2}}{6\pi c}\left(\frac{x}{b}\right)^{6}\left(\frac{c^{2}b^{2}}{x^{3}}\right)^{2} && (0.8)\\
&= \frac{\mu_{0}q^{2}c^{3}}{6\pi b^{2}} && (0.9)
\end{aligned}$$

Thus the radiated power is constant as the charge moves along its trajectory.

The radiation reaction force is given by

$$F_{rad} = \frac{\mu_0 q^2 \gamma^4}{6\pi c}\left(\dot{a} + \frac{3a^2\gamma^2 v}{c^2}\right) \tag{0.10}$$

for which we need

$$\dot{a} = -\frac{3c^4 b^2 t}{x^5} \tag{0.11}$$

We get

$$\frac{3a^2\gamma^2 v}{c^2} = \frac{3}{c^2}\left(\frac{c^2 b^2}{x^3}\right)^2\left(\frac{x}{b}\right)^2\left(\frac{tc^2}{x}\right) \tag{0.12}$$

$$= \frac{3c^4 b^2 t}{x^5} \tag{0.13}$$

$$= -\dot{a} \tag{0.14}$$

$$F_{rad} = 0 \tag{0.15}$$

This is a rather curious case in that although the charge radiates, there is no radiation reaction force.