INERTIAL FRAMES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 1-2.

Although we've already looked at special relativity in earlier posts, I think it's a good idea to revisit the theory by following the treatment in Griffiths's book in his Chapter 12. The reason is that Griffiths's book develops the theory specifically with a view to showing how it applies to classical electromagnetism, which is something we haven't really looked at before.

Central to both classical and relativistic mechanics is the notion of an *inertial frame of reference*. Griffiths's definition of an inertial frame is one of the simplest that I've seen. Basically, an inertial frame is a frame in which Newton's first law holds. Newton's first law states that, in the absence of an external force, an object travels in a straight line at constant speed (or just at constant velocity, since velocity incorporates both speed and direction of travel).

Example 1. If $\mathcal{S}$ is an inertial frame, then if another frame $\bar{\mathcal{S}}$ moves at constant velocity $\mathbf{v}$ relative to $\mathcal{S}$, it too is an inertial frame. To prove this, we need to specify whether we are using Galilean or relativistic mechanics. In Galilean mechanics, the velocity addition rule is that if an object's velocity as measured in $\mathcal{S}$ is $\mathbf{v}_1$ then its velocity as measured in $\bar{\mathcal{S}}$ is

$$\bar{\mathbf{v}}_1 = \mathbf{v}_1 - \mathbf{v} \tag{1}$$

If both $\mathbf{v}$ and $\mathbf{v}_1$ are constant, then so too is $\bar{\mathbf{v}}_1$, so $\bar{\mathcal{S}}$ must be an inertial frame.

In relativistic mechanics, the velocity addition formula is (if $\mathbf{v}$ and $\mathbf{v}_1$ are parallel):

$$\bar{v}_1 = \frac{v_1 - v}{1 - v_1 v / c^2} \tag{2}$$

and again, if v_1 and v are constants, so too is $\bar{v}_1$ so $\bar{\mathcal{S}}$ is an inertial frame. [If $\mathbf{v}$ and $\mathbf{v}_1$ are not parallel, this formula applies to the components of the velocities that are parallel. The perpendicular components are the same in both frames, so they too must be constant.]

Conversely, if $\bar{\mathcal{S}}$ and $\mathcal{S}$ are both inertial systems, then the velocities of an object with no external forces acting on it must be constant in both frames, which implies that the relative velocity $\mathbf{v}$ must be a constant, as can be seen from either of the formulas above.

Example 2. Considering classical (Galilean) mechanics only, suppose that in an inertial frame $\mathcal{S}$ we have a collision between a particle A (mass m_A and velocity $\mathbf{u}_A$) and another particle B (mass m_B and velocity $\mathbf{u}_B$). During the collision, some of the mass of A gets transferred to B, so that afterwards we have particles C and D with masses m_C, m_D and velocities $\mathbf{u}_C, \mathbf{u}_D$. If momentum is conserved, then

$$m_A\mathbf{u}_A + m_B\mathbf{u}_B = m_C\mathbf{u}_C + m_D\mathbf{u}_D \tag{3}$$

$$m_A + m_B = m_C + m_D \tag{4}$$

In inertial frame $\bar{\mathcal{S}}$ which moves with velocity $\mathbf{v}$ relative to $\mathcal{S}$, the velocities of the particles transform as

$$\bar{\mathbf{u}}_i = \mathbf{u}_i - \mathbf{v} \tag{5}$$

The conservation of momentum equation is then

$$m_A(\bar{\mathbf{u}}_A + \mathbf{v}) + m_B(\bar{\mathbf{u}}_B + \mathbf{v}) = m_C(\bar{\mathbf{u}}_C + \mathbf{v}) + m_D(\bar{\mathbf{u}}_D + \mathbf{v}) \tag{6}$$

$$m_A\bar{\mathbf{u}}_A + m_B\bar{\mathbf{u}}_B + (m_A + m_B)\mathbf{v} = m_C\bar{\mathbf{u}}_C + m_D\bar{\mathbf{u}}_D + (m_C + m_D)\mathbf{v} \tag{7}$$

$$m_A\bar{\mathbf{u}}_A + m_B\bar{\mathbf{u}}_B = m_C\bar{\mathbf{u}}_C + m_D\bar{\mathbf{u}}_D \tag{8}$$

where to get the last line, we assumed that mass is conserved as in 4.

An elastic collision is one in which the total kinetic energy is conserved. [In an inelastic collision, momentum is conserved but kinetic energy is transformed into another form of energy, such as heat. If two objects of equal mass are travelling towards each other with the same speed and upon collision, they stick together and come to rest, then the total momentum is zero before and after the collision, but the kinetic energy has been reduced to zero.] If the collision is elastic in $\mathcal{S}$, then

$$\frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}m_C u_C^2 + \frac{1}{2}m_D u_D^2 \tag{9}$$

$$m_A\left(\bar{u}_A^2 + v^2 + 2\bar{\mathbf{u}}_A\cdot\mathbf{v}\right) + m_B\left(\bar{u}_B^2 + v^2 + 2\bar{\mathbf{u}}_B\cdot\mathbf{v}\right) = m_C\left(\bar{u}_C^2 + v^2 + 2\bar{\mathbf{u}}_C\cdot\mathbf{v}\right) + m_D\left(\bar{u}_D^2 + v^2\right. \tag{10}$$

$$m_A\bar{u}_A^2 + m_B\bar{u}_B^2 + (m_A + m_B)v^2 + 2(m_A\bar{\mathbf{u}}_A + m_B\bar{\mathbf{u}}_B)\cdot\mathbf{v} = m_C\bar{u}_C^2 + m_D\bar{u}_D^2 + (m_C + m_D)v^2 + 2(m \tag{11}$$

We can cancel the middle terms on both sides using 4 and the last terms on both sides using 8, so we're left with

$$m_A\bar{u}_A^2+m_B\bar{u}_B^2=m_C\bar{u}_C^2+m_D\bar{u}_D^2 \tag{12}$$

so the collision is elastic in $\bar{S}$ as well.

PINGBACKS

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