

VELOCITY ADDITION IN SPECIAL RELATIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 12.3.

The velocity addition formula in special relativity is (if \mathbf{v}_a and \mathbf{v}_b are parallel):

$$(0.1) \quad v_r = \frac{v_a + v_b}{1 + v_a v_b / c^2}$$

Example 1. To get a feel for how small the correction to the classical formula $v_c = v_a + v_b$ is for everyday speeds, suppose $v_a = 5$ miles/hour = 2.235 m s^{-1} and $v_b = 60$ miles/hour = 26.82 m s^{-1} . Since $v_a v_b / c^2$ in this case is very small, we can approximate v_r by

$$(0.2) \quad v_r \approx (v_a + v_b) \left(1 - \frac{v_a v_b}{c^2} \right)$$

The percentage error in the classical formula is then

$$(0.3) \quad \Delta v = \frac{v_c - v_r}{v_c} \times 100\%$$

$$(0.4) \quad \approx \frac{(v_a + v_b) (v_a v_b / c^2)}{v_a + v_b} \times 100\%$$

$$(0.5) \quad = \frac{v_a v_b}{c^2} \times 100\%$$

$$(0.6) \quad = \frac{2.235 \times 26.82}{(3 \times 10^8)^2} \times 100\%$$

$$(0.7) \quad = 6.66 \times 10^{-14}\%$$

It's not surprising that no relativistic effects are seen in the everyday world.

Example 2. Suppose you could run at $v_a = 0.5c$ (relative to the train) down the corridor of a train travelling at $v_b = 0.75c$. An observer on the ground would see your speed relative to the ground as

$$(0.8) \quad v = \frac{0.5 + 0.75}{1 + (0.5)(0.75)}c$$

$$(0.9) \quad = 0.91c$$

Even though the classical sum of velocities is greater than c , the relativistic formula still gives a result that is less than c .

Example 3. The formula 0.1 always gives a result that is less than c . To prove this, we can simplify the notation by using velocities that are fractions of c so that $a \equiv v_a/c$ and $b \equiv v_b/c$ with the sum given by $s \equiv v_r/c$. Then

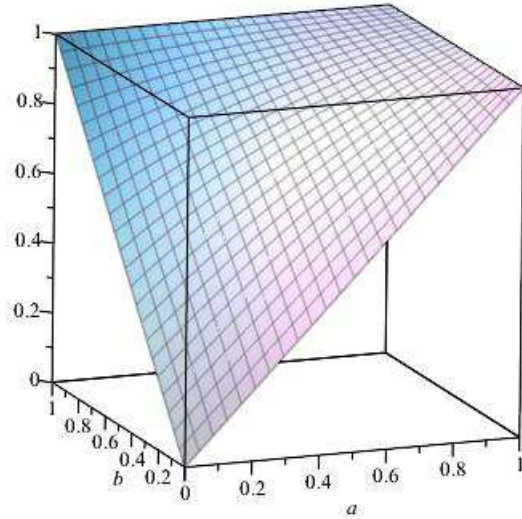
$$(0.10) \quad s = \frac{a + b}{1 + ab}$$

To check if there are any extrema in the region $0 \leq a \leq 1$, $0 \leq b \leq 1$ we can take the two partial derivatives and set them to zero:

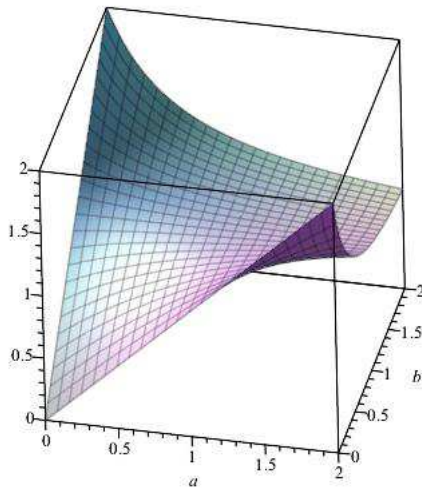
$$(0.11) \quad \frac{\partial s}{\partial a} = \frac{1}{1 + ab} - \frac{(a + b)b}{(1 + ab)^2} = 0$$

$$(0.12) \quad \frac{\partial s}{\partial b} = \frac{1}{1 + ab} - \frac{(a + b)a}{(1 + ab)^2} = 0$$

The only solution within the region is $a = b = 1$. We can see that this must be a maximum within the region, since along the border $a = 1$ or the border $b = 1$ we have $s = 1$, along the border $a = 0$ we have $s = b$ and along the border $b = 0$ we have $s = a$ so that $s \leq 1$ on all borders. Thus $s \leq 1$ everywhere in the region $0 \leq a \leq 1$, $0 \leq b \leq 1$. The surface 0.10 within this region looks like this:



The point $a = b = 1$ is, however, actually a saddle point, as an expanded plot of the region $0 \leq a \leq 2$, $0 \leq b \leq 2$ reveals:



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