APPARENT SPEEDS GREATER THAN THE SPEED OF LIGHT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 6.

Although no object can travel faster than light, it is possible for the *apparent* speed of an object to be greater than c. A common example is that of the apparent motion of a star across the sky. Suppose a star is a distance $a = L$ from Earth at time t_a and that its velocity $\mathbf{v}$ is towards Earth at an angle θ to the line of sight. At time t_b it arrives at a point b whose distance from Earth is

$$b = L - v\cos\theta\,(t_b - t_a) \tag{1}$$

The times of arrival at Earth of the light emitted at distances a and b are

$$\begin{aligned} T_a &= t_a + \frac{L}{c} && (2)\\ T_b &= t_b + \frac{1}{c}\left[L - v\cos\theta\,(t_b - t_a)\right] && (3) \end{aligned}$$

During this time, the star moves a distance perpendicular to the line of sight of $v\sin\theta\,(t_b - t_a)$, so the apparent speed as seen from Earth is

$$\begin{aligned} u &= \frac{v\sin\theta\,(t_b - t_a)}{T_b - T_a} && (4)\\ &= \frac{v\sin\theta\,(t_b - t_a)}{(t_b - t_a)\left(1 - \frac{v}{c}\cos\theta\right)} && (5)\\ &= \frac{v\sin\theta}{1 - \frac{v}{c}\cos\theta} && (6) \end{aligned}$$

This speed has a maximum at an angle θ which can be found by setting the derivative to zero:

$$\frac{du}{d\theta} = v\cos(\theta)\left(1-\frac{v\cos(\theta)}{c}\right)^{-1} - \frac{v^2(\sin(\theta))^2}{c}\left(1-\frac{v\cos(\theta)}{c}\right)^{-2} \quad (7)$$

$$= \frac{(\cos(\theta)c - v)cv}{(\cos(\theta))^2 v^2 - 2c\cos(\theta)v + c^2} \quad (8)$$

$$= 0 \quad (9)$$

$$\cos\theta = \frac{v}{c} \quad (10)$$

At this angle, the apparent speed is

$$u_{max} = \frac{v\sqrt{1-v^2/c^2}}{1-v^2/c^2} \quad (11)$$

$$= \gamma v \quad (12)$$

Since $\gamma \to \infty$ as $v \to c$, u_{max} can be much larger than c even though the *actual* speed of the star is less than c. This again illustrates the importance of correctly interpreting the raw data that we see, and of allowing for the travel time of the light.

COMMENTS

Remark 1. Posted by incomprehensiblething on 25 April 2018, 12:56.

I believe your statement that "no object can travel faster than light" is misleading. There is a much cited paper by Davis and Lineweaver (2003) called "Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe". In it they state: "there is no contradiction with special relativity when faster than light motion occurs outside the observer's inertial frame" and "it is well-accepted that general relativity, not special relativity, is necessary to describe cosmological observations." The first misconception they discuss is that "recession velocities cannot exceed the speed of light". Apparently they can. And not only can they but we can also (counter-intuitively) observe galaxies with superluminal velocities. All objects with redshift greater than about 1.46 are are receding faster than the speed of light.

Remark 2. Posted by incomprehensiblething on 27 April 2018, 9:23.

See also Ben Crowell's comments here. In particular, he says: "For one fairly popular definition of the velocity (based on distances measured by rulers at rest with respect to the Hubble flow), we can actually observe galaxies that are moving away from us at >c, and that always have been moving away from us at >c."