APPARENT SPEEDS GREATER THAN THE SPEED OF LIGHT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 6.

Although no object can travel faster than light, it is possible for the *apparent* speed of an object to be greater than c. A common example is that of the apparent motion of a star across the sky. Suppose a star is a distance $a=L$ from Earth at time t_a and that its velocity $\mathbf{v}$ is towards Earth at an angle θ to the line of sight. At time t_b it arrives at a point b whose distance from Earth is

$$b=L-v\cos\theta\,(t_b-t_a) \tag{1}$$

The times of arrival at Earth of the light emitted at distances a and b are

$$\begin{aligned} T_a &= t_a+\frac{L}{c} && (2)\\ T_b &= t_b+\frac{1}{c}\left[L-v\cos\theta\,(t_b-t_a)\right] && (3) \end{aligned}$$

During this time, the star moves a distance perpendicular to the line of sight of $v\sin\theta\,(t_b-t_a)$, so the apparent speed as seen from Earth is

$$\begin{aligned} u &= \frac{v\sin\theta\,(t_b-t_a)}{T_b-T_a} && (4)\\ &= \frac{v\sin\theta\,(t_b-t_a)}{(t_b-t_a)\left(1-\frac{v}{c}\cos\theta\right)} && (5)\\ &= \frac{v\sin\theta}{1-\frac{v}{c}\cos\theta} && (6) \end{aligned}$$

This speed has a maximum at an angle θ which can be found by setting the derivative to zero:

$$\frac{du}{d\theta} = v\cos(\theta)\left(1-\frac{v\cos(\theta)}{c}\right)^{-1} - \frac{v^2(\sin(\theta))^2}{c}\left(1-\frac{v\cos(\theta)}{c}\right)^{-2} \tag{7}$$

$$= \frac{(\cos(\theta)\,c - v)\,cv}{(\cos(\theta))^2 v^2 - 2c\cos(\theta)\,v + c^2} \tag{8}$$

$$= 0 \tag{9}$$

(10)

$$\cos\theta = \frac{v}{c}$$

At this angle, the apparent speed is

$$u_{max} = \frac{v\sqrt{1-v^2/c^2}}{1-v^2/c^2} \tag{11}$$

$$= \gamma v \tag{12}$$

Since $\gamma \to \infty$ as $v \to c$, u_{max} can be much larger than c even though the *actual* speed of the star is less than c. This again illustrates the importance of correctly interpreting the raw data that we see, and of allowing for the travel time of the light.