

APPARENT SPEEDS GREATER THAN THE SPEED OF LIGHT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 6.

Although no object can travel faster than light, it is possible for the *apparent* speed of an object to be greater than c . A common example is that of the apparent motion of a star across the sky. Suppose a star is a distance $a = L$ from Earth at time t_a and that its velocity \mathbf{v} is towards Earth at an angle θ to the line of sight. At time t_b it arrives at a point b whose distance from Earth is

$$(0.1) \quad b = L - v \cos \theta (t_b - t_a)$$

The times of arrival at Earth of the light emitted at distances a and b are

$$(0.2) \quad T_a = t_a + \frac{L}{c}$$

$$(0.3) \quad T_b = t_b + \frac{1}{c} [L - v \cos \theta (t_b - t_a)]$$

During this time, the star moves a distance perpendicular to the line of sight of $v \sin \theta (t_b - t_a)$, so the apparent speed as seen from Earth is

$$(0.4) \quad u = \frac{v \sin \theta (t_b - t_a)}{T_b - T_a}$$

$$(0.5) \quad = \frac{v \sin \theta (t_b - t_a)}{(t_b - t_a) \left(1 - \frac{v}{c} \cos \theta\right)}$$

$$(0.6) \quad = \frac{v \sin \theta}{1 - \frac{v}{c} \cos \theta}$$

This speed has a maximum at an angle θ which can be found by setting the derivative to zero:

(0.7)

$$\frac{du}{d\theta} = v \cos(\theta) \left(1 - \frac{v \cos(\theta)}{c}\right)^{-1} - \frac{v^2 (\sin(\theta))^2}{c} \left(1 - \frac{v \cos(\theta)}{c}\right)^{-2}$$

$$(0.8) \quad = \frac{(\cos(\theta) c - v) cv}{(\cos(\theta))^2 v^2 - 2c \cos(\theta) v + c^2}$$

$$(0.9) \quad = 0$$

(0.10)

$$\cos \theta = \frac{v}{c}$$

At this angle, the apparent speed is

$$(0.11) \quad u_{max} = \frac{v \sqrt{1 - v^2/c^2}}{1 - v^2/c^2}$$

$$(0.12) \quad = \gamma v$$

Since $\gamma \rightarrow \infty$ as $v \rightarrow c$, u_{max} can be much larger than c even though the *actual* speed of the star is less than c . This again illustrates the importance of correctly interpreting the raw data that we see, and of allowing for the travel time of the light.