

APPARENT SPEEDS GREATER THAN THE SPEED OF LIGHT

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 6.

Although no object can travel faster than light, it is possible for the *apparent* speed of an object to be greater than c . A common example is that of the apparent motion of a star across the sky. Suppose a star is a distance $a = L$ from Earth at time t_a and that its velocity \mathbf{v} is towards Earth at an angle θ to the line of sight. At time t_b it arrives at a point b whose distance from Earth is

$$b = L - v \cos \theta (t_b - t_a) \quad (1)$$

The times of arrival at Earth of the light emitted at distances a and b are

$$T_a = t_a + \frac{L}{c} \quad (2)$$

$$T_b = t_b + \frac{1}{c} [L - v \cos \theta (t_b - t_a)] \quad (3)$$

During this time, the star moves a distance perpendicular to the line of sight of $v \sin \theta (t_b - t_a)$, so the apparent speed as seen from Earth is

$$u = \frac{v \sin \theta (t_b - t_a)}{T_b - T_a} \quad (4)$$

$$= \frac{v \sin \theta (t_b - t_a)}{(t_b - t_a) \left(1 - \frac{v}{c} \cos \theta\right)} \quad (5)$$

$$= \frac{v \sin \theta}{1 - \frac{v}{c} \cos \theta} \quad (6)$$

This speed has a maximum at an angle θ which can be found by setting the derivative to zero:

$$\frac{du}{d\theta} = v \cos(\theta) \left(1 - \frac{v \cos(\theta)}{c}\right)^{-1} - \frac{v^2 (\sin(\theta))^2}{c} \left(1 - \frac{v \cos(\theta)}{c}\right)^{-2} \quad (7)$$

$$= \frac{(\cos(\theta)c - v)cv}{(\cos(\theta))^2 v^2 - 2c \cos(\theta)v + c^2} \quad (8)$$

$$= 0 \quad (9)$$

$$\cos \theta = \frac{v}{c} \quad (10)$$

At this angle, the apparent speed is

$$u_{max} = \frac{v\sqrt{1 - v^2/c^2}}{1 - v^2/c^2} \quad (11)$$

$$= \gamma v \quad (12)$$

Since $\gamma \rightarrow \infty$ as $v \rightarrow c$, u_{max} can be much larger than c even though the *actual* speed of the star is less than c . This again illustrates the importance of correctly interpreting the raw data that we see, and of allowing for the travel time of the light.