TIME DILATION: RESOLVING THE PARADOX

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 7.

Using our usual setup of a train moving with velocity v next to an observer standing beside the track, suppose we have a light bulb on the ceiling of the train which is turned on at $t=0$. If the height of the train is h, then to observer T on the train, the light takes a time $\Delta t_T = h/c$ to reach the floor, since the light just travels straight down from ceiling to floor. To the observer G on the ground, the light travels in a diagonal path due to the train's motion, so if it takes time Δt_G to reach the floor, this time is determined by

$$\Delta t_G = \frac{1}{c}\sqrt{h^2 + (v\Delta t_G)^2} \tag{1}$$

Solving for Δt_G:

$$\Delta t_G = \frac{h}{c\sqrt{1-v^2/c^2}} \tag{2}$$

[Note that we've implicitly assumed that the height of the train is measured to be h to both observers. We'll return to this point when we consider length contraction.]

Since $\Delta t_T = h/c$, we get the *time dilation* formula:

$$\Delta t_G = \gamma \Delta t_T \tag{3}$$

$$\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}} \tag{4}$$

That is $\Delta t_G > \Delta t_T$, a principle that is often stated as "moving clocks run slow". Another way of putting it is that the proper time is always the minimum time measured between two events. The *proper time* is the time measured by an observer moving with the clock being used to make the time measurements. In the case of the train, the proper time is Δt_T since this is the time interval measured by a clock moving with the train.

Example. A popular demonstration of time dilation is the effect it has on the lifetime of unstable particles, such as the muon which has a half-life of 2×10^{-6} s when it is at rest. If the muon is travelling at some speed v, its lifetime measured by proper time is still 2×10^{-6} s, since the proper time clock is moving with the muon. To an observer in the lab, however, the elapsed time is longer by a factor of γ. Suppose a muon is observed in the lab to move 800 m from the time of its creation to its decay (and that its proper time lifetime is equal to the half-life). In that case, its lab lifetime is

$$\Delta t_L = 2\times10^{-6}\gamma \tag{5}$$

and is speed is given by

$$\begin{aligned} v &= \frac{800\text{ m}}{2\times10^{-6}\gamma} && (6)\\ &= 4\times10^{8}\sqrt{1-\frac{v^2}{c^2}} && (7)\\ v &= 2.4\times10^{8}\text{ m s}^{-1} && (8)\end{aligned}$$

If we hadn't taken time dilation into account its velocity would be $v = 800/2\times10^{-6} = 4\times10^{8}$ m s^{-1} which is greater than c.

Newcomers to relativity often get confused by the apparent contradiction with time dilation. After all, if the train clock runs slow relative to the ground clock, then the ground clock must run fast relative to the train clock. But to an observer on the train, it is the ground clock that is moving, so *it* should be running slow relative to the train clock.

This apparent paradox arises because what we are actually measuring in the above experiment with the light beam isn't symmetric between the two observers. Observer T on the train uses only one clock to measure both the start and end points of the light's journey from ceiling to floor. Observer G on the ground, however, must use *two* clocks, one of which is situated at the point where the light leaves the ceiling and the second at a point down the line where the light reaches the floor. As these two clocks are stationary relative to each other, observer G can synchronize them so that the times registered on the two clocks will always agree. However, as we've seen, simultaneity is relative, so two events that appear synchronous to G will not appear synchronous to T. Thus T disputes G's claim that his clocks are synchronized, which results in the difference in time measurements.

Another way of looking at it is this. Suppose we now have two trains, one of which is stationary (the G train) and the other (the T train) which moves with speed v relative to G. This time, *both* trains run the experiment of firing a light beam from the ceiling to the floor. As we've seen above, observer G thinks that it takes a longer time for the light beam on T to reach the floor than T does. By the same logic, T will think it takes longer for G's light beam to reach the floor of G than G does.

T says that it takes time h/c for his own light beam to reach his own floor. However, if G fires his light beam at the same time as T (in other words, both G and T define their respective time origins by the simultaneous firing of their light beams when the two trains are at the same location), T says that, since G's beam has further to travel, by the time T's beam has hit the floor, G's beam hasn't yet completed its journey. According to G, his own beam takes time h/c to reach the floor, so if his own beam hasn't yet reached the floor, G's time must be less than h/c, showing that G's clock runs slow relative to T's clock. The important point is that the clock that runs slow in both cases is the one that is fixed relative to the respective train. Thus G's fixed clock runs slow relative to the two clocks that T uses to measure the travel time of G's light beam, and T 's fixed clock runs slow relative to the two clocks that G uses to measure the travel time of T 's light beam. The fact that both experiments involve comparing one clock in one reference frame to *two* clocks in the other reference frame shows that the theory is consistent and there is no paradox.

PINGBACKS

Pingback: Time dilation: a rocket passing Earth
Pingback: Lorentz (length) contraction: a simple example
Pingback: Lorentz transformations: forward and backward
Pingback: The twin paradox analyzed using Lorentz transformations
Pingback: Four-velocity again
Pingback: The light clock and time dilation
Pingback: Length contraction and time dilation: a few examples
Pingback: Another trip to Alpha Centauri: more Lorentz transformation examples