TIME DILATION: A ROCKET PASSING EARTH

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 8.

As an example of time dilation, suppose a rocket going at $v=\frac{3}{5}c$ passes Earth at $t=0$ (Griffiths's statement of the problem is that the rocket 'leaves Earth' at that time, but that would involve acceleration, so to be picky, I'll just have the rocket going at a constant speed passing Earth at $t=0$). One hour later, as measured on the rocket, the rocket sends a light signal back to Earth. According to clocks in Earth's frame, at what time did the rocket send the signal?

As always in time dilation calculations, we need to identify the frame in which the clock measures proper time, that is, the frame in which the clock is fixed to the moving object. In this case, the 1 hour measured by the rocket's clock is the rocket's proper time, and the time in Earth's frame is measured by *two* clocks, one on Earth when the rocket passes by, and one beside the rocket when it sends the signal. Therefore in this case, it is the rocket's clock that appears to run slow, so the time measured by Earth will be longer than 1 hour.

$$t_E = \gamma t_R \tag{1}$$

$$= \frac{1\text{ hour}}{\sqrt{1-v^2/c^2}} \tag{2}$$

$$= \frac{5}{4}\text{ hours} \tag{3}$$

Now we want to find the time, as measured by Earth, that the signal arrives at Earth. Here we can do all calculations in Earth's frame. Since the signal left the rocket at $t=\frac{5}{4}$ hours and the rocket is travelling at $\frac{3}{5}c$ it was at a distance of $d=\frac{5}{4}\frac{3}{5}c=\frac{3}{4}c$ light hours. Therefore it takes the signal $\frac{3}{4}$ hour to arrive back on Earth, so it will arrive at $t=\frac{5}{4}+\frac{3}{4}=2$ hours.

How long after the rocket left Earth did the signal arrive on Earth, according to an observer on the rocket? This time, we are looking at two events that both occur on Earth, so the 2 hours (as measured by Earth) that have elapsed on Earth between the rocket passing by and the signal arriving is the proper time, so the time measured by the rocket will be longer than this.

To measure the time in the rocket's frame, the rocket will have to use *two* clocks, one at the location of the Earth when the rocket passes it, and the other at the location of the Earth when the signal arrives. Therefore

$$\begin{aligned} t_R &= \gamma t_E && (4)\\ &= \frac{5}{4}\times 2\text{ hours} && (5)\\ &= \frac{5}{2}\text{ hours} && (6) \end{aligned}$$