LORENTZ CONTRACTION: NO CONTRACTION IN DIRECTIONS PERPENDICULAR TO THE MOTION

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 10.

We've seen that the length of a moving object in the direction of relative motion is contracted by the factor γ. What about lengths perpendicular to the relative motion? It turns out that these lengths are unaffected by the relative motion.

I've never seen a mathematical derivation of this result. Most books just state it as though it were obvious, but Griffiths quotes a thought experiment originally due to E. F. Taylor and J. A. Wheeler in 1966. Suppose we have our usual moving train, with an observer T on the train and another observer G on the ground. Next to the train is a wall, and before the train arrives, G paints a horizontal red line on the wall at a distance of 1 m above the ground, as measured by G. Then, as the train goes by, T leans out the window and paints a blue line at a distance of 1 m above the ground, as measured by T. After the train passes by, there are two lines painted on the wall. Which is higher?

If there were some sort of contraction effect for distances perpendicular to the motion, then G would say that T should perceive his 1 m above the ground to be less than G's 1 m, so the blue line should be below the red one. However, to T, it is G that is moving so it is G that should perceive *his* 1 m to be less than T's 1 m, so the red line should be lower. As these two results are contradictory, the only rational conclusion is that both T and G measure 1 m to be exactly the same distance, and the blue and red lines are at exactly the same height.

This argument might bother you, since it seems similar to arguments that time dilation and length contraction (in the direction of motion) are contradictory. If T's clock moves relative to G's, then T's clock must run slow, but to T, it is G's clock that is moving and so should be running slow. Similarly, for length contraction, each observer sees the other's lengths as being contracted. However, in both of these situations, the apparent paradox is resolved by taking into account the discrepancy in simultaneity between the two observers, and in both these cases, the disagreements relate to events that occur at specific coordinates in both frames and that leave no lasting effect.

In the case of painting lines on the wall, the two lines remain on the wall after the experiment is finished and, if there *were* a contraction for either observer, the two lines would have to be at different heights on the wall. No jiggery-pokery with simultaneity can change that. The fact that assuming a contraction in the perpendicular direction leads to a contradiction shows that lengths perpendicular to the motion are unchanged.

Example. Suppose a sailboat moves at speed v relative to an observer on the shore. The sailboat has a mast of length L that is anchored near the front of the boat and makes an angle (when the boat is at rest) of θ with the deck of the boat. What angle will the observer on the shore see?

Since only the component of the mast parallel to the motion is contracted, the shore observer will see an angle θ' that satisfies:

$$\frac{L\sin\theta}{\frac{1}{\gamma}L\cos\theta} = \frac{L\sin\theta'}{L\cos\theta'} \tag{1}$$

$$\tan\theta' = \gamma\tan\theta \tag{2}$$

The faster the boat goes, the larger is γ, so the apparent angle will increase towards $\pi/2$. This makes sense, since the horizontal component gets contracted while the vertical component remains the same.

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