

LORENTZ TRANSFORMATIONS: FORWARD AND BACKWARD

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 12.

We can combine the effects of time dilation and length contraction to derive the Lorentz transformations. As usual, we align the two inertial frames \mathcal{S} and $\bar{\mathcal{S}}$ (which moves to the right at speed v relative to \mathcal{S}) so that the x and \bar{x} axes coincide at their origins also coincide at $t = \bar{t} = 0$. Consider a point on the \bar{x} axis (that, obviously, has coordinate \bar{x} in the $\bar{\mathcal{S}}$ system). Due to length contraction, the distance from the origin of $\bar{\mathcal{S}}$ to \bar{x} as measured in \mathcal{S} is (remember that the measurement of a distance in \mathcal{S} means that both endpoints of the distance are measured at the same time t , and that these two measurement events in \mathcal{S} will be seen as occurring at *different* times \bar{t}_1 and \bar{t}_2 in the $\bar{\mathcal{S}}$ system):

$$d = \frac{\bar{x}}{\gamma} \quad (1)$$

so, since $\bar{\mathcal{S}}$ is moving at speed v relative to \mathcal{S} , the location of point \bar{x} as measured by \mathcal{S} is

$$x = d + vt \quad (2)$$

$$= \frac{\bar{x}}{\gamma} + vt \quad (3)$$

$$\bar{x} = \gamma(x - vt) \quad (4)$$

Obviously, we can invert the procedure by looking at the point of view of $\bar{\mathcal{S}}$. In this case, everything is the same, except that \mathcal{S} is moving to the *left* with speed $-v$, so

$$x = \gamma(\bar{x} + v\bar{t}) \quad (5)$$

We can use this equation to eliminate \bar{x} from 4 to get

$$\frac{x}{\gamma} - v\bar{t} = \gamma(x - vt) \quad (6)$$

$$\bar{t} = \frac{1}{v} \left[\frac{x}{\gamma} - \gamma(x - vt) \right] \quad (7)$$

$$= \frac{\gamma}{v} \left[x \left(1 - \frac{v^2}{c^2} - 1 \right) + vt \right] \quad (8)$$

$$= \gamma \left(t - \frac{xv}{c^2} \right) \quad (9)$$

Since directions perpendicular to the motion don't change, the full set of Lorentz transformations is

$$\bar{x} = \gamma(x - vt) \quad (10)$$

$$\bar{y} = y \quad (11)$$

$$\bar{z} = z \quad (12)$$

$$\bar{t} = \gamma \left(t - \frac{xv}{c^2} \right) \quad (13)$$

As mentioned above, it's fairly obvious that the inverse transformations can be obtained by replacing v by $-v$:

$$x = \gamma(\bar{x} + v\bar{t}) \quad (14)$$

$$y = \bar{y} \quad (15)$$

$$z = \bar{z} \quad (16)$$

$$t = \gamma \left(\bar{t} + \frac{\bar{x}v}{c^2} \right) \quad (17)$$

If you must, you can derive these from the original transformations. Starting with 4 and then 13:

$$x = \frac{\bar{x}}{\gamma} + vt \quad (18)$$

$$\bar{t} = \gamma \left(t - \frac{v\bar{x}}{\gamma c^2} - \frac{v^2}{c^2} t \right) \quad (19)$$

$$= \gamma \left(\frac{t}{\gamma^2} - \frac{v\bar{x}}{\gamma c^2} \right) \quad (20)$$

$$= \frac{t}{\gamma} - \frac{v\bar{x}}{c^2} \quad (21)$$

$$t = \gamma \left(\bar{t} + \frac{\bar{x}v}{c^2} \right) \quad (22)$$

Starting with 13 and then 4

$$t = \frac{\bar{t}}{\gamma} + \frac{v\bar{x}}{c^2} \quad (23)$$

$$\bar{x} = \gamma \left(x - \frac{v\bar{t}}{\gamma} - \frac{v^2}{c^2} x \right) \quad (24)$$

$$= \gamma \left(\frac{x}{\gamma^2} - \frac{v\bar{t}}{\gamma} \right) \quad (25)$$

$$x = \gamma (\bar{x} + v\bar{t}) \quad (26)$$

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