LORENTZ TRANSFORMATIONS: FORWARD AND BACKWARD

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 12.

We can combine the effects of time dilation and length contraction to derive the Lorentz transformations. As usual, we align the two inertial frames $\mathscr{S}$ and $\bar{\mathscr{S}}$ (which moves to the right at speed v relative to $\mathscr{S}$) so that the x and $\bar{x}$ axes coincide at their origins also coincide at $t=\bar{t}=0$. Consider a point on the $\bar{x}$ axis (that, obviously, has coordinate $\bar{x}$ in the $\bar{\mathscr{S}}$ system). Due to length contraction, the distance from the origin of $\bar{\mathscr{S}}$ to $\bar{x}$ as measured in $\mathscr{S}$ is (remember that the measurement of a distance in $\mathscr{S}$ means that both endpoints of the distance are measured at the same time t, and that these two measurement events in $\mathscr{S}$ will be seen as occurring at *different* times $\bar{t}_1$ and $\bar{t}_2$ in the $\bar{\mathscr{S}}$ system):

$$d=\frac{\bar{x}}{\gamma} \tag{0.1}$$

so, since $\bar{\mathscr{S}}$ is moving at speed v relative to $\mathscr{S}$, the location of point $\bar{x}$ as measured by $\mathscr{S}$ is

$$\begin{aligned}
x &= d+vt && (0.2)\\
&= \frac{\bar{x}}{\gamma}+vt && (0.3)\\
\bar{x} &= \gamma(x-vt) && (0.4)
\end{aligned}$$

Obviously, we can invert the procedure by looking at the point of view of $\bar{\mathscr{S}}$. In this case, everything is the same, except that $\mathscr{S}$ is moving to the *left* with speed $-v$, so

$$x=\gamma(\bar{x}+v\bar{t}) \tag{0.5}$$

We can use this equation to eliminate $\bar{x}$ from 0.4 to get

$$\frac{x}{\gamma} - v\bar{t} = \gamma(x - vt) \tag{0.6}$$

$$\bar{t} = \frac{1}{v}\left[\frac{x}{\gamma} - \gamma(x - vt)\right] \tag{0.7}$$

$$= \frac{\gamma}{v}\left[x\left(1 - \frac{v^2}{c^2} - 1\right) + vt\right] \tag{0.8}$$

$$= \gamma\left(t - \frac{xv}{c^2}\right) \tag{0.9}$$

Since directions perpendicular to the motion don't change, the full set of Lorentz transformations is

$$\bar{x} = \gamma(x - vt) \tag{0.10}$$

$$\bar{y} = y \tag{0.11}$$

$$\bar{z} = z \tag{0.12}$$

$$\bar{t} = \gamma\left(t - \frac{xv}{c^2}\right) \tag{0.13}$$

As mentioned above, it's fairly obvious that the inverse transformations can be obtained by replacing v by $-v$:

$$x = \gamma(\bar{x} + v\bar{t}) \tag{0.14}$$

$$y = \bar{y} \tag{0.15}$$

$$z = \bar{z} \tag{0.16}$$

$$t = \gamma\left(\bar{t} + \frac{\bar{x}v}{c^2}\right) \tag{0.17}$$

If you must, you can derive these from the original transformations. Starting with 0.4 and then 0.13:

$$x = \frac{\bar{x}}{\gamma}+vt \tag{0.18}$$

$$\bar{t} = \gamma\left(t-\frac{v\bar{x}}{\gamma c^{2}}-\frac{v^{2}}{c^{2}}t\right) \tag{0.19}$$

$$= \gamma\left(\frac{t}{\gamma^{2}}-\frac{v\bar{x}}{\gamma c^{2}}\right) \tag{0.20}$$

$$= \frac{t}{\gamma}-\frac{v\bar{x}}{c^{2}} \tag{0.21}$$

$$t = \gamma\left(\bar{t}+\frac{\bar{x}v}{c^{2}}\right) \tag{0.22}$$

Starting with 0.13 and then 0.4

$$t = \frac{\bar{t}}{\gamma}+\frac{vx}{c^{2}} \tag{0.23}$$

$$\bar{x} = \gamma\left(x-\frac{v\bar{t}}{\gamma}-\frac{v^{2}}{c^{2}}x\right) \tag{0.24}$$

$$= \gamma\left(\frac{x}{\gamma^{2}}-\frac{v\bar{t}}{\gamma}\right) \tag{0.25}$$

$$x = \gamma(\bar{x}+v\bar{t}) \tag{0.26}$$

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