

LORENTZ TRANSFORMATIONS: FORWARD AND BACKWARD

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 12.

We can combine the effects of time dilation and length contraction to derive the Lorentz transformations. As usual, we align the two inertial frames \mathcal{S} and $\bar{\mathcal{S}}$ (which moves to the right at speed v relative to \mathcal{S}) so that the x and \bar{x} axes coincide at their origins also coincide at $t = \bar{t} = 0$. Consider a point on the \bar{x} axis (that, obviously, has coordinate \bar{x} in the $\bar{\mathcal{S}}$ system). Due to length contraction, the distance from the origin of $\bar{\mathcal{S}}$ to \bar{x} as measured in \mathcal{S} is (remember that the measurement of a distance in \mathcal{S} means that both endpoints of the distance are measured at the same time t , and that these two measurement events in \mathcal{S} will be seen as occurring at *different* times \bar{t}_1 and \bar{t}_2 in the $\bar{\mathcal{S}}$ system):

$$(1) \quad d = \frac{\bar{x}}{\gamma}$$

so, since $\bar{\mathcal{S}}$ is moving at speed v relative to \mathcal{S} , the location of point \bar{x} as measured by \mathcal{S} is

$$(2) \quad x = d + vt$$

$$(3) \quad = \frac{\bar{x}}{\gamma} + vt$$

$$(4) \quad \bar{x} = \gamma(x - vt)$$

Obviously, we can invert the procedure by looking at the point of view of $\bar{\mathcal{S}}$. In this case, everything is the same, except that \mathcal{S} is moving to the *left* with speed $-v$, so

$$(5) \quad x = \gamma(\bar{x} + v\bar{t})$$

We can use this equation to eliminate \bar{x} from 4 to get

$$\begin{aligned}
 (6) \quad \frac{x}{\gamma} - v\bar{t} &= \gamma(x - vt) \\
 (7) \quad \bar{t} &= \frac{1}{v} \left[\frac{x}{\gamma} - \gamma(x - vt) \right] \\
 (8) \quad &= \frac{\gamma}{v} \left[x \left(1 - \frac{v^2}{c^2} - 1 \right) + vt \right] \\
 (9) \quad &= \gamma \left(t - \frac{xv}{c^2} \right)
 \end{aligned}$$

Since directions perpendicular to the motion don't change, the full set of Lorentz transformations is

$$\begin{aligned}
 (10) \quad \bar{x} &= \gamma(x - vt) \\
 (11) \quad \bar{y} &= y \\
 (12) \quad \bar{z} &= z \\
 (13) \quad \bar{t} &= \gamma \left(t - \frac{xv}{c^2} \right)
 \end{aligned}$$

As mentioned above, it's fairly obvious that the inverse transformations can be obtained by replacing v by $-v$:

$$\begin{aligned}
 (14) \quad x &= \gamma(\bar{x} + v\bar{t}) \\
 (15) \quad y &= \bar{y} \\
 (16) \quad z &= \bar{z} \\
 (17) \quad t &= \gamma \left(\bar{t} + \frac{\bar{x}v}{c^2} \right)
 \end{aligned}$$

If you must, you can derive these from the original transformations. Starting with 4 and then 13:

$$(18) \quad x = \frac{\bar{x}}{\gamma} + vt$$

$$(19) \quad \bar{t} = \gamma \left(t - \frac{v\bar{x}}{\gamma c^2} - \frac{v^2}{c^2} t \right)$$

$$(20) \quad = \gamma \left(\frac{t}{\gamma^2} - \frac{v\bar{x}}{\gamma c^2} \right)$$

$$(21) \quad = \frac{t}{\gamma} - \frac{v\bar{x}}{c^2}$$

$$(22) \quad t = \gamma \left(\bar{t} + \frac{\bar{x}v}{c^2} \right)$$

Starting with 13 and then 4

$$(23) \quad t = \frac{\bar{t}}{\gamma} + \frac{v\bar{x}}{c^2}$$

$$(24) \quad \bar{x} = \gamma \left(x - \frac{v\bar{t}}{\gamma} - \frac{v^2}{c^2} x \right)$$

$$(25) \quad = \gamma \left(\frac{x}{\gamma^2} - \frac{v\bar{t}}{\gamma} \right)$$

$$(26) \quad x = \gamma(\bar{x} + v\bar{t})$$

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