VELOCITY ADDITION: CHASING SPACE PIRATES VIEWED IN FOUR REFERENCE FRAMES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 15.

As an example of using the relativistic velocity addition formula, we can revisit the problem of the space police chasing some pirates. In the problem, space pirates are fleeing from the solar system police in a spacecraft that is moving at $\frac{3}{4}c$ (relative to the Earth). The police's spaceship is travelling at only $\frac{1}{2}c$ but in an attempt to stop the pirates they fire a torpedo at them. The torpedo's velocity, relative to the police's ship, is $\frac{1}{3}c$.

Originally, we worked out the velocity of the torpedo relative to Earth as $\frac{5}{7}c$ and saw that the torpedo would not catch the pirates. We can use the velocity addition formula to work out the velocity of each component in the problem relative to every other component and check that the pirates escape no matter how we look at it. If an object has velocity $\bar{u}$ in system $\bar{\mathcal{S}}$ which is moving at speed v relative to system $\mathcal{S}$, then the object's velocity u as measured in $\mathcal{S}$ is

$$u = \frac{\bar{u}+v}{1+\bar{u}v/c^2} \tag{1}$$

The velocity of the torpedo relative to the Earth is

$$v_r = \frac{\frac{1}{2}c+\frac{1}{3}c}{1+\frac{1}{6}} = \frac{5}{7}c = \frac{20}{28}c \tag{2}$$

The speed v_{pp} of the pirates relative to the police is found from the condition that the velocity v_{pe} of the pirates relative to Earth is

$$v_{pe} = \frac{v_{pp}+\frac{1}{2}c}{1+\left(\frac{1}{2}c\right)v_{pp}/c^2} \tag{3}$$

Solving for v_{pp} we get

$$
\begin{aligned}
v_{pp} &= \frac{v_{pe} - \frac{1}{2}c}{1 - \left(\frac{1}{2}c\right) v_{pe}/c^2} && (4) \\
&= \frac{\frac{3}{4}c - \frac{1}{2}c}{1 - \left(\frac{1}{2}c\right)\left(\frac{3}{4}c\right)/c^2} && (5) \\
&= \frac{2}{5}c && (6)
\end{aligned}
$$

Similarly, the velocity v_{tp} of the torpedo relative to the pirates is

$$
\begin{aligned}
v_{tp} &= \frac{\frac{5}{7}c - \frac{3}{4}c}{1 - \left(\frac{5}{7}c\right)\left(\frac{3}{4}c\right)/c^2} && (7) \\
&= -\frac{1}{13}c && (8)
\end{aligned}
$$

In summary, the following matrix gives the velocity of the object named in the column header relative to the object in the row header:

	Earth	Police	Pirates	Torpedo	Escape?
Earth	0	$\frac{1}{2}c$	$\frac{3}{4}c$	$\frac{5}{7}c$	Yes
Police	$-\frac{1}{2}c$	0	$\frac{2}{5}c$	$\frac{1}{3}c$	Yes
Pirates	$-\frac{3}{4}c$	$-\frac{2}{5}c$	0	$-\frac{1}{13}c$	Yes
Torpedo	$-\frac{5}{7}c$	$-\frac{1}{3}c$	$\frac{1}{13}c$	0	Yes

The matrix is antisymmetric ($A_{ij} = -A_{ji}$) since the velocity of A relative to B is just the negative of the velocity of B relative to A. In all cases, $v_{torpedo} < v_{pirates}$ so the pirates always escape.