

## INVARIANCE OF SCALAR PRODUCT UNDER LORENTZ TRANSFORMATIONS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.17.

Reference: Carroll, Bradley W. & Ostlie, Dale A. (2007), *An Introduction to Modern Astrophysics*, 2nd Edition; Pearson Education - Chapter 4, Problem 4.11.

Although the time and position of an event can change under Lorentz transformations, the scalar product of any two four-vectors is an invariant under a Lorentz transformation. That is

$$(1) \quad \bar{a}_i \bar{b}^i = a_i b^i$$

where for motion along the 1-axis ( $x$  axis) the transformations are

$$(2) \quad \bar{a}^0 = \gamma(a^0 - \beta a^1)$$

$$(3) \quad \bar{a}^1 = \gamma(a^1 - \beta a^0)$$

$$(4) \quad \bar{a}^2 = a^2$$

$$(5) \quad \bar{a}^3 = a^3$$

and

$$(6) \quad a_0 = -\bar{a}^0$$

$$(7) \quad a_j = \bar{a}^j$$

for  $j = 1, 2, 3$ . We can see this directly by calculation, using  $\gamma^2 = 1/(1 - \beta^2)$ :

(8)

$$\bar{a}_i \bar{b}^i = -\gamma^2 (a^0 - \beta a^1) (b^0 - \beta b^1) + \gamma^2 (a^1 - \beta a^0) (b^1 - \beta b^0) + a^2 b^2 + a^3 b^3$$

(9)

$$= \gamma^2 [a^0 b^0 (-1 + \beta^2) + a^1 b^0 (\beta - \beta) + a^0 b^1 (\beta - \beta) + a^1 b^1 (-\beta^2 + 1)] + a^2 b^2 + a^3 b^3$$

(10)

$$= -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$$

(11)

$$= a_i b^i$$

In particular if  $a^i$  is the space-time four-vector of an event, or the difference between the four-vectors of two events, then

(12)

$$\bar{a}_i \bar{a}^i = a_i a^i$$

This leads to the *invariant interval* between two events:

(13)

$$\Delta s^2 \equiv -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

#### PINGBACKS

Pingback: The invariant interval: some examples