RAPIDITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 19.

An alternative way of writing the Lorentz transformations is to define a quantity called the *rapidity*:

$$\theta \equiv \tanh^{-1}\beta \tag{1}$$

Using this definition, we have

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{2}$$

$$= \frac{1}{\sqrt{1-\tanh^2\theta}} \tag{3}$$

$$= \frac{\cosh\theta}{\sqrt{\cosh^2\theta-\sinh^2\theta}} \tag{4}$$

$$= \cosh\theta \tag{5}$$

since $\cosh^2\theta-\sinh^2\theta=1$.

Also

$$\gamma\beta = \cosh\theta\tanh\theta = \sinh\theta \tag{6}$$

so

$$\Lambda = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

This is similar to a rotation through an angle θ in 3-d space, except both the sinh terms are negative.

The velocity addition formula becomes

$$\bar{u} = \frac{u+v}{1+uv/c^2} \tag{9}$$

$$= \frac{\beta_u+\beta_v}{1+\beta_u\beta_v}c \tag{10}$$

$$\beta_{\bar{u}} = \frac{\tanh\theta_u+\tanh\theta_v}{1+\tanh\theta_u\tanh\theta_v} \tag{11}$$

$$\tanh\theta_{\bar{u}} = \tanh\left(\theta_u+\theta_v\right) \tag{12}$$

where in the last line we've used the formula for the tanh of a sum of two arguments.

The rapidities therefore simply add, giving a simpler measure of relativistic velocity:

$$\theta_{\bar{u}} = \theta_u+\theta_v \tag{13}$$

PINGBACKS

Pingback: Four-velocity again