RAPIDITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 19.

An alternative way of writing the Lorentz transformations is to define a quantity called the *rapidity*:

$$\theta \equiv \tanh^{-1}\beta \tag{1}$$

Using this definition, we have

$$\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-\beta^2}} &(2)\\
&= \frac{1}{\sqrt{1-\tanh^2\theta}} &(3)\\
&= \frac{\cosh\theta}{\sqrt{\cosh^2\theta-\sinh^2\theta}} &(4)\\
&= \cosh\theta &(5)
\end{aligned}$$

since $\cosh^2\theta-\sinh^2\theta=1$.

Also

$$\gamma\beta = \cosh\theta\tanh\theta = \sinh\theta \tag{6}$$

so

$$\begin{aligned}
\Lambda &= \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} &(7)\\
&= \begin{bmatrix} \cosh\theta & -\sinh\theta & 0 & 0\\ -\sinh\theta & \cosh\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} &(8)
\end{aligned}$$

This is similar to a rotation through an angle θ in 3-d space, except both the sinh terms are negative.

The velocity addition formula becomes

$$\begin{aligned}
\bar{u} &= \frac{u+v}{1+uv/c^{2}} && (9)\\
&= \frac{\beta_{u}+\beta_{v}}{1+\beta_{u}\beta_{v}}c && (10)\\
\beta_{\bar{u}} &= \frac{\tanh\theta_{u}+\tanh\theta_{v}}{1+\tanh\theta_{u}\tanh\theta_{v}} && (11)\\
\tanh\theta_{\bar{u}} &= \tanh\left(\theta_{u}+\theta_{v}\right) && (12)
\end{aligned}$$

where in the last line we've used the formula for the tanh of a sum of two arguments.

The rapidities therefore simply add, giving a simpler measure of relativistic velocity:

$$\theta_{\bar{u}}=\theta_{u}+\theta_{v} \tag{13}$$

PINGBACKS

Pingback: Four-velocity again