SPACELIKE INTERVALS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 22.

If two events are separated by a spacelike interval, neither event can affect the other, since different observers may disagree about the order of the events. Here are examples of a couple of misconceptions that sometimes arise about such intervals.

Example 1. If two people are sitting a couple of metres apart then they are at rest relative to each other so they share the same inertial frame and will agree about all time and space measurements. At a particular instant of time, the interval separating the two people is spacelike, so it might seem that they could not communicate with each other. However, if we draw each person's world line on a spacetime diagram, then in their own frame, each person's world line is a vertical line (remember that ct is plotted on the ordinate ('y axis') and x on the abscissa ('x axis'). If they are speaking to each other, the sound waves have world lines that travel diagonally upwards between the two vertical world lines representing the 2 people. If person A is at $x=0$ and B is at $x=2$, then if A says something at $t=0$, the sound reaches B at $t=2/v$ where v is the speed of sound, so the slope of the sound wave's world line is

$$c\frac{\Delta t}{\Delta x}=c\frac{2}{2v}=\frac{c}{v} \tag{1}$$

As $c\gg v$, the sound's world line is very nearly vertical but it does angle from A's vertical line over to B's line. Similarly, if B says something to A, the sound's world line travels in the $-x$ direction with the same speed, so its slope is $-c/v$.

The interval between the events of A saying something and B hearing it is

$$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 \tag{2}$$

$$= -c^2\frac{4}{v^2} + 4 \tag{3}$$

$$= -4\frac{c^2}{v^2}\left(1 - \frac{v^2}{c^2}\right) \tag{4}$$

For $c \gg v$, this is (a very large) negative value, so the interval between the two events is definitely timelike.

Example 2. Suppose that faster than light travel *is* possible, but that light signals still travel at c. In that case it would be possible for an object to travel from A to B such that the interval between the events of leaving A and arriving at B is spacelike. Since different observers can disagree on the order in which such events occur, it is possible for some observers to say that the object arrived at B *before* it left A.

However, if the object then returned from B to A (also faster than light, say), all observers would agree that the object arrived back at A after it left A. This is because the interval between the two events (leaving A and arriving back at A) is timelike (since they occur at the same place in A's frame), so they must be separated by a positive time interval in every inertial frame.