

SPACETIME DIAGRAMS: AN EXAMPLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 23.

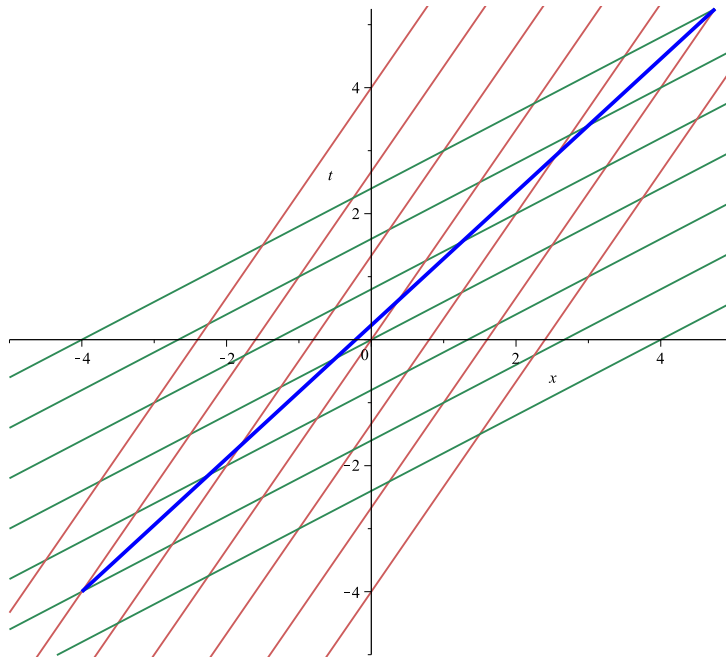
As an example of a spacetime diagram, suppose we have our usual two inertial frames with \mathcal{S} at rest relative to observer A and $\bar{\mathcal{S}}$ moving at speed $\beta = \frac{3}{5}$ in the $+x$ direction, with the origins of the two systems coinciding as usual.

We'd like to plot the lines of constant \bar{t} and \bar{x} on a spacetime diagram. Using Lorentz transformations we have

$$ct = \frac{x}{\beta} - \frac{\bar{x}}{\beta\gamma} \quad (1)$$

$$ct = \beta x + \frac{c\bar{t}}{\gamma} \quad (2)$$

For various values of \bar{x} and \bar{t} , these two equations give two sets of parallel lines. The first equation gives lines with slope $1/\beta$ and ct -intercepts of $-\bar{x}/\beta\gamma$, while the second equation gives lines with slope β and ct -intercepts of $c\bar{t}/\gamma$. We can plot a few lines from each set as shown:



The red lines are lines of constant \bar{x} with the top line corresponding to $\bar{x} = -3$ and the bottom line to $\bar{x} = +3$, in steps of 1. The green lines are lines of constant \bar{t} with the bottom line corresponding to $\bar{t} = -3$ and the top line to $\bar{t} = +3$, again in steps of 1.

The thick blue line represents the world line of an object that starts at $(\bar{t}, \bar{x}) = (-2, -2)$ and moves to $(\bar{t}, \bar{x}) = (3, 2)$. We can find its velocity in \mathcal{S} by taking its slope on the graph. Finding the exact values of x and t is difficult by eyeballing a graph, so we can 'cheat' a bit and use the Lorentz transformations to find the corresponding values. We get for the starting point:

$$ct_1 = \gamma(ct + \beta\bar{x}) \quad (3)$$

$$= \frac{5}{4} \left(-2 - \frac{3}{5}2 \right) \quad (4)$$

$$= -4 \quad (5)$$

$$x_1 = \gamma(\bar{x} + \beta ct) \quad (6)$$

$$= -4 \quad (7)$$

and for the end point:

$$ct_2 = \gamma(ct\bar{t} + \beta\bar{x}) \quad (8)$$

$$= \frac{5}{4} \left(3 + \frac{3}{5} 2 \right) \quad (9)$$

$$= \frac{21}{4} \quad (10)$$

$$x_2 = \gamma(\bar{x} + \beta ct\bar{t}) \quad (11)$$

$$= \frac{19}{4} \quad (12)$$

The velocity is then

$$v = \frac{\Delta x}{\Delta t} \quad (13)$$

$$= \frac{35}{37} c \quad (14)$$

We can check this using the velocity addition formula. Its velocity in \bar{S} is

$$\bar{v} = \frac{\Delta \bar{x}}{\Delta \bar{t}} = \frac{4}{5} c \quad (15)$$

so

$$v = \frac{\frac{4}{5} + \frac{3}{5}}{1 + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)} c = \frac{35}{37} c \quad (16)$$

so it checks out.