

SPACETIME DIAGRAMS: AN EXAMPLE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 23.

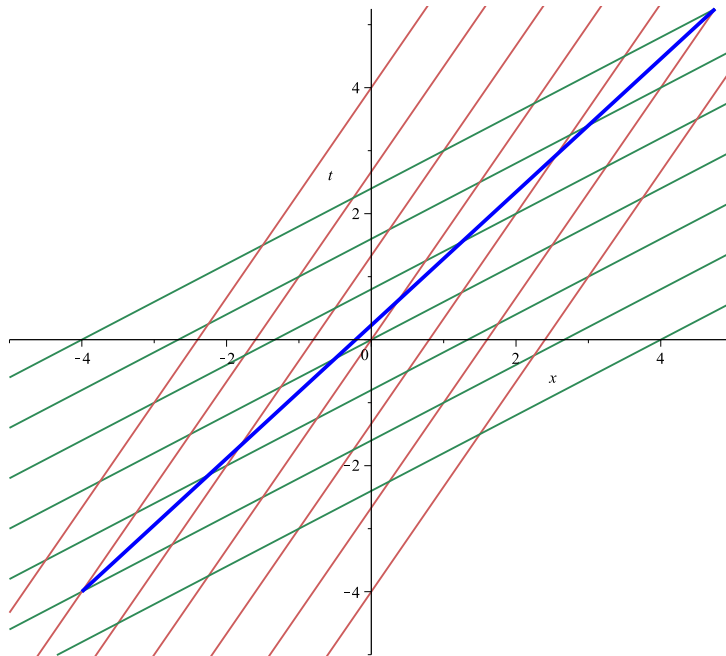
As an example of a spacetime diagram, suppose we have our usual two inertial frames with \mathcal{S} at rest relative to observer A and $\bar{\mathcal{S}}$ moving at speed $\beta = \frac{3}{5}$ in the $+x$ direction, with the origins of the two systems coinciding as usual.

We'd like to plot the lines of constant \bar{t} and \bar{x} on a spacetime diagram. Using Lorentz transformations we have

$$(1) \quad ct = \frac{x}{\beta} - \frac{\bar{x}}{\beta\gamma}$$

$$(2) \quad ct = \beta x + \frac{c\bar{t}}{\gamma}$$

For various values of \bar{x} and \bar{t} , these two equations give two sets of parallel lines. The first equation gives lines with slope $1/\beta$ and ct -intercepts of $-\bar{x}/\beta\gamma$, while the second equation gives lines with slope β and ct -intercepts of $c\bar{t}/\gamma$. We can plot a few lines from each set as shown:



The red lines are lines of constant \bar{x} with the top line corresponding to $\bar{x} = -3$ and the bottom line to $\bar{x} = +3$, in steps of 1. The green lines are lines of constant \bar{t} with the bottom line corresponding to $\bar{t} = -3$ and the top line to $\bar{t} = +3$, again in steps of 1.

The thick blue line represents the world line of an object that starts at $(\bar{t}, \bar{x}) = (-2, -2)$ and moves to $(\bar{t}, \bar{x}) = (3, 2)$. We can find its velocity in \mathcal{S} by taking its slope on the graph. Finding the exact values of x and t is difficult by eyeballing a graph, so we can 'cheat' a bit and use the Lorentz transformations to find the corresponding values. We get for the starting point:

$$\begin{aligned}
 (3) \quad ct_1 &= \gamma(c\bar{t} + \beta\bar{x}) \\
 (4) \quad &= \frac{5}{4} \left(-2 - \frac{3}{5}2 \right) \\
 (5) \quad &= -4 \\
 (6) \quad x_1 &= \gamma(\bar{x} + \beta c\bar{t}) \\
 (7) \quad &= -4
 \end{aligned}$$

and for the end point:

$$\begin{aligned}
 (8) \quad ct_2 &= \gamma(ct\bar{+} + \beta\bar{x}) \\
 (9) \quad &= \frac{5}{4} \left(3 + \frac{3}{5} 2 \right) \\
 (10) \quad &= \frac{21}{4} \\
 (11) \quad x_2 &= \gamma(\bar{x} + \beta ct\bar{+}) \\
 (12) \quad &= \frac{19}{4}
 \end{aligned}$$

The velocity is then

$$\begin{aligned}
 (13) \quad v &= \frac{\Delta x}{\Delta t} \\
 (14) \quad &= \frac{35}{37} c
 \end{aligned}$$

We can check this using the velocity addition formula. Its velocity in $\bar{\mathcal{S}}$ is

$$(15) \quad \bar{v} = \frac{\Delta \bar{x}}{\Delta \bar{t}} = \frac{4}{5} c$$

so

$$(16) \quad v = \frac{\frac{4}{5} + \frac{3}{5}}{1 + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)} c = \frac{35}{37} c$$

so it checks out.