

FOUR-VELOCITY'S SQUARE IS A UNIVERSAL CONSTANT

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 26.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c = 1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.

The four velocity is defined as

$$(0.1) \quad \eta^i = \frac{u^i}{\sqrt{1 - u^2/c^2}}$$

so the square is

$$(0.2) \quad \eta_i \eta^i = -\frac{c^2}{1 - u^2/c^2} + \frac{\sum_{i=1}^3 (u^i)^2}{1 - u^2/c^2}$$

$$(0.3) \quad = \frac{-c^2 + u^2}{1 - u^2/c^2}$$

$$(0.4) \quad = -c^2$$

This is the equivalent of the result we derived earlier in Moore's book, except with $c = 1$ so $\eta_i \eta^i = -1$. It is a universal invariant for all four-velocities.

In terms of four-momentum, this relation becomes

$$(0.5) \quad p_i p^i = -m^2 c^2$$

$$(0.6) \quad -(p^0)^2 + p^2 = -m^2 c^2$$

$$(0.7) \quad -(cp^0)^2 + c^2 p^2 = -m^2 c^4$$

$$(0.8) \quad E^2 - p^2 c^2 = m^2 c^4$$

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