FOUR-VELOCITY'S SQUARE IS A UNIVERSAL CONSTANT

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 26.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c=1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.

The four velocity is defined as

$$\eta^{i}=\frac{u^{i}}{\sqrt{1-u^{2}/c^{2}}} \tag{1}$$

so the square is

$$\begin{aligned} \eta_{i}\eta^{i} &= -\frac{c^{2}}{1-u^{2}/c^{2}}+\frac{\sum_{i=1}^{3}\left(u^{i}\right)^{2}}{1-u^{2}/c^{2}} &(2)\\ &= \frac{-c^{2}+u^{2}}{1-u^{2}/c^{2}} &(3)\\ &= -c^{2} &(4)\end{aligned}$$

This is the equivalent of the result we derived earlier in Moore's book, except with $c=1$ so $\eta_{i}\eta^{i}=-1$. It is a universal invariant for all four-velocities.

In terms of four-momentum, this relation becomes

$$\begin{aligned} p_{i}p^{i} &= -m^{2}c^{2} &(5)\\ -\left(p^{0}\right)^{2}+p^{2} &= -m^{2}c^{2} &(6)\\ -\left(cp^{0}\right)^{2}+c^{2}p^{2} &= -m^{2}c^{4} &(7)\\ E^{2}-p^{2}c^{2} &= m^{2}c^{4} &(8)\end{aligned}$$

PINGBACKS

Pingback: Decay of a pion into a muon and a neutrino

Pingback: Decay of a pion into two photons
Pingback: Relative energy in a particle collider
Pingback: Pair annihilation with an electron and a positron