CENTRE OF MOMENTUM FRAME

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 30.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c = 1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.]

In classical mechanics, it's often easier to analyze the motions of objects in their centre of mass frame. In relativity, it's more accurate to refer to the *centre of momentum* frame, which is the frame in which the sums of the spatial components of the momentum of all objects are zero. The problem is to find this frame given the energies and momenta of a set of objects.

In some inertial frame, the total energy and momentum of such a set are (assuming all motion is along the x axis):

$$E_{tot} = \sum E_i = \sum p_i^0 c \tag{1}$$

$$P_{tot} = \sum p_i = \sum p_i^1 \tag{2}$$

Using Lorentz transformations, we want a frame moving with velocity v relative to this frame such that

$$\bar{P}_{tot} = 0 \tag{3}$$

Therefore

$$\bar{P}_{tot} = \gamma \sum \left(p_i^1 - \beta p_i^0\right) = 0 \tag{4}$$

$$\beta = \frac{\sum p_i^1}{\sum p_i^0} \tag{5}$$

$$v = c^2 \frac{\sum p_i}{\sum E_i} \tag{6}$$

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